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Magnetic models of slowly rotating magnetic Ap stars: aligned magnetic and rotation axes

J. D. Landstreet[1] and G. Mathys[2]

[1] University of Western Ontario, Department of Physics & Astronomy, London, Ontario, Canada N6A 3K7
email: jlandstr@uwo.ca
[2] European Southern Observatory, Casilla 19001, Santiago 19, Chile
email: gmathys@eso.org

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Abstract. As a result of major surveys carried out during the past decade by Mathys and collaborators, we now have measurements with full phase coverage of several magnetic field moments, including the mean longitudinal field B_ℓ, the mean field modulus $B_{\rm s}$, and in most cases the mean quadratic field $B_{\rm mq}$ and mean crossover field $B_{\rm xover}$, for a sample of 24 chemically peculiar magnetic (Ap) stars. This represents an increase of a factor of order five in the stellar sample with data of this quality, compared to the situation a decade ago.

We exploit this dataset to derive general and statistical properties of the stars in the sample, as follows. First, we fit the available field moment observations assuming a simple, axisymmetric multipole magnetic field expansion (with dipole, quadrupole, and octupole components) over each stellar surface. We show that this representation, though not exact, gives an adequate description of the available data for all the stars in this sample, although the fit parameters are in many cases not unique. We find that many of the stars require an important quadrupole and/or octupole field component to satisfy the observations, and that some (usually small) deviations from our assumed axisymmetric field distributions are certainly present. We examine the inclination i $(0 \leq i \leq 90°)$ of the rotation axis to the line of sight and the obliquity β $(0 \leq \beta \leq 90°)$ of the magnetic field with respect to the rotation axis, and show that the stars with periods of the order of a month or longer have systematically small values of β: *slowly rotating magnetic stars generally have their magnetic and rotation axes aligned to within about 20°, unlike the short period magnetic Ap stars, in which β is usually large.* This is a qualitatively new result, and one which is very important for efforts to understand the evolution of magnetic fields and angular momentum in the magnetic Ap stars.

Key words: Stars, chemically peculiar – Stars, magnetic fields – Stars, fundamental properties – Stars, statistics

Send offprint requests to: J. D. Landstreet

1. Introduction

It has been known from more than half a century (Babcock 1947) that a significant fraction of main sequence A and B stars have strong, globally ordered magnetic fields. The fields of these magnetic Ap and Bp stars are usually detected by the observation of circular polarization in the wings of spectral line profiles due to the Zeeman effect (e.g. Landstreet 1980). The first (wavelength) moment of the circular polarization with respect to the line centre is proportional to the amplitude of the mean of the line-of-sight component $\langle B_z \rangle$ of the magnetic field on the visible stellar hemisphere, and the surface moment of the magnetic field thus measured is known as the mean longitudinal field, B_ℓ (e.g. Landstreet 1982, Mathys 1989). For most magnetic A and B stars the observed B_ℓ varies approximately sinusoidally through the rotation period of the star (e.g. Bohlender et al. 1993, Mathys & Hubrig 1997), indicating that the stellar field has a fairly simple overall structure (or else mean line-of-sight field component would essentially average to zero, as it does on the sun), and that this magnetic structure is in general not axisymmetric about the stellar rotation axis.

It has long been of interest to deduce from measurements of the stellar magnetic field as a function of rotational phase what the underlying distribution of the local magnetic field vector over the stellar surface is. This is usually done by assuming a parameterized form for the field distribution over the surface (e.g. a multipole expansion), calculating the resulting mean longitudinal field and its variation with stellar rotation, and varying the parameters of the model until agreement with the observed variation of B_ℓ is reached. It is found that sinusoidal variation of B_ℓ is consistent with a simple global dipole field distribution in which the dipole is centred in the star and its axis is inclined at an angle β to the rotation axis. In this case, the axis of the dipole sweeps out a cone around the stellar rotation axis as the star rotates, the inclination of the dipole axis to the line of sight changes with time, and the observer sees a sinusoidal variation of the mean line-of-sight field component.

However, the observation of the variation of B_ℓ is not sufficient to yield a unique set of parameters even for such a simple model field geometry as an oblique dipole. The observations of B_ℓ yields essentially two parameters (say, the maximum and minimum observed field values, or the mean and semi-amplitude of the sinusoidal variation), while even this simplest model requires three parameters (the inclination i of the rotation axis to the line of sight, the obliquity β of the field axis to the rotation axis, and the polar strength B_d of the dipole field). Because none of these parameters is in general well known independently (although if the stellar radius R, the projected rotational velocity $v \sin i$, and the period P are all known fairly precisely, the value of i may be deduced from $v \sin i = 2\pi R \sin i / P$), dipole field models are usually not unique. The fact that even the simplest meaningful model is not uniquely constrained by the most widely used method of measuring the stellar field, and that no further information about the field structure is available from such measurements except in the unusual case where the variation of B_ℓ is *not* sinusoidal (e.g. Landstreet 1990) has been an ongoing difficulty in modelling stellar fields.

Further information about the stellar field geometry may be obtained by observing other structure introduced into line profiles by the Zeeman effect in the presence of a field. This can be achieved by studying Zeeman structure in the line profile in intensity (Babcock 1960), in circular polarization (Semel 1989), or (very recently) in linear polarization (Wade et al. 2000), or by observing the integrated broad-band linear polarization that is also produced by the Zeeman effect (Landolfi et al. 1993; Leroy et al. 1996). If the spectral lines are quite narrow (typically with $v \sin i$ of a few km s^{-1} or less), it may be possible to observe the actual splitting of some lines into the π and σ Zeeman components. This splitting is proportional to the average of the local field modulus $\langle B \rangle$ over the visible hemisphere, and provides a measurement of a field moment called the mean surface field or mean field modulus B_s (Mathys 1990). For somewhat larger values of $v \sin i$ it is still possible to detect the broadening effect of the field on the line profile and to deduce a related quantity (from consideration of the second wavelength moment of the intensity line profile) which is approximately proportional to the quantity $(\langle B^2 \rangle + \langle B_z^2 \rangle)^{1/2}$. This field moment is called the mean quadratic field B_mq (Mathys 1995b). The second wavelength moment of the circular polarization line profiles provides a measurement of the field moment called the crossover field B_xover, which is proportional to the disk mean $v \sin i \langle x B_z \rangle$, where x is the normalized distance from the stellar rotation axis in the plane of the sky (Mathys 1995a).

Ten years ago, the observational situation for magnetic Ap stars was that B_ℓ data were available for at least 50 stars (e.g. Didelon 1983), while B_s observations through a full stellar rotation were available for just four stars. For these four stars, the combination of B_ℓ and B_s data (which together provide at a minimum *four* constraints on the model) made it possible to deduce almost unique field geometry models with both a dipole and a parallel linear quadrupole (e.g. Landstreet 1980). An important result was that in all four stars a significant quadrupole component is required in addition to the expected dipole, which is equivalent to saying that the field strength at one stellar magnetic pole is stronger than at the other. Furthermore, modelling of actual line profiles for two of these stars (Landstreet 1988, Landstreet et al. 1989) showed that an octupole component was also required to model the fields; a simple dipole has too much contrast between the field at the poles and at the equator to fit the data for these two stars. These results demonstrate clearly that the observation of further field moments beyond B_ℓ provide much valuable information about stellar field geometries.

In the past decade, a major observational survey programme has been undertaken by Mathys and a number of collaborators to obtain measurements, through the full rotation cycles of several dozen stars, of the field moments B_ℓ, B_s, B_mq, and B_xover. The most recent publications in this series, from which references to earlier measurements can be found, are Mathys & Hubrig (1997) for B_ℓ, B_mq, and B_xover, and Mathys et al. (1997) for B_s. As a result of this work, we now have available reasonably complete magnetic curves of at least B_ℓ and B_s, and in most cases also B_mq and B_xover, for about 20 stars. This survey has led to an increase in the available sample of stars for which well-constrained field geometry models can be derived of about a factor of five. This is such a large increase in the sample size that it is now possible not only to study the field structures of individual stars (which is already being done, e.g. Wade et al. 1996; Bagnulo et al. 1999), but also to examine the statistical behaviour of an interestingly large sample of magnetic Ap stars to see what general features emerge from homogeneous modelling of the full currently available ensemble. One such characteristic of these sharp-line stars that has already been discovered in the observations is the – as yet unexplained – fact that 10 of 16 stars with periods shorter than 150 days have surface fields B_s that exceed 7.5 kG, while none of the 19 stars with longer periods have $B_\mathrm{s} > 7.5$ kG (Mathys et al 1997). We hope to find other statistical regularities in this greatly enlarged sample.

In this paper, we have two goals. First, we derive preliminary models for all of the stars for which observations of both B_ℓ and B_s (the two observational moments whose relationship to the underlying field distribution seems most clear) are available, in order to examine in particular how well the observations may be fit with the oblique three-term axisymmetric multipole expansion (parallel dipole, quadrupole, and octupole) that has been fairly successful in previous modelling, in order to learn whether such a field distribution constitutes a reasonable first approximation for the field structure of a large ensemble of stars, and to what extent unique parameter sets can be derived for each star. We also examine the question of how well such simple parallel multipole models fit the newly available observations of $B_{\rm mq}$ and $B_{\rm xover}$, and try to understand what new information these data bring to modelling, and how consistent they are with more traditional kinds of data. Secondly, we explore the regularities that are present in the derived field models when the whole ensemble of modelled stars is examined.

In the next section, we discuss the modelling strategy used to find best-fit field models for the observations. In the following section, we derive and discuss models for the individual stars. The statistical properties of the ensemble of models are then discussed, and the paper closes with some general remarks and conclusions.

2. Modelling strategy

Given the inclination i of the rotation axis, the obliquity β of the magnetic axis, and the specification of a magnetic field geometry (for example by setting the polar values for the field dipole $B_{\rm d}$, quadrupole $B_{\rm q}$, and octupole $B_{\rm oct}$ components in a three-term axisymmetric multipole expansion), as well as a specification of the relative weighting of various parts of the visible stellar hemisphere, it is straightforward to compute the various integrated field moments such as B_ℓ, $B_{\rm s}$, etc. observed as a function of rotational phase. However, the inverse problem is not so easily solved. In principle one could construct a programme that reads in all the observations (and errors) of each field moment with its phase, and use a method such as the conjugate gradient technique to search the parameter space of possible model field geometries by successive forward calculations of field moments for a best fit to the observations. However, construction of such a programme is rather costly in time and effort, and a much simpler numerical approach has been found to be sufficient for this first modelling of field geometries with the assumed multipole expansion.

Rather than reading in the full data set of observations for each observed field moment, we take advantage of the fact that generally each field moment is observed to vary in a smooth and fairly simple way with rotational phase. A smooth curve is put through the observations of each field moment and values are read off at four equally spaced standard phases separated by 0.25 cycles, starting from the phase of closest approach of a magnetic pole to the line of sight, which is assumed to be the phase of largest absolute value of B_ℓ. (This is found to be enough phases to lead to computed curves in reasonable overall agreement with the data; with fewer points the computed curves sometimes agree with the observations only at the fitting points.) For each type of data (B_ℓ, $B_{\rm s}$, etc.), a single standard deviation is assigned which represents a typical value of the uncertainty of that type of measurement for the star being modelled; varying this quantity allows one to weight the various field moments more or less heavily.

The programme reads in these data, and from them makes a first estimate of likely values of the polar values of the multipole component. The initial value of $B_{\rm d}$ is set to 1.5 times the maximum value of $B_{\rm s}$, while the initial value of $B_{\rm q}$ is set to 0, as was the value of $B_{\rm oct}$ at first. (The initial multipole field strengths can also be chosen manually to start with some other values.) The programme then calculates the expected variations of the various observable field moments B_ℓ, $B_{\rm s}$, and $B_{\rm mq}$ over the full i, β plane (using a grid point spacing of 10°) and compares them with the four values at the standard phases, computing a reduced χ^2 for the fit. This process is repeated for a series of five values of each field strength parameter ($B_{\rm d}$, etc.) around the initial values, with initial step size of $0.25 B_{\rm d}$ in all field values. The value of the reduced χ^2 is computed for each parameter set, and the minimum value, and the corresponding field and geometrical parameters are recorded. The values of i, β, $B_{\rm d}$, etc. that led to the lowest value of χ^2/ν are then taken as the centre of a new small grid of five values in each parameter, and the exploration of the grid is repeated centred on this new point in parameter space. From this step on, exploration of i and β is treated the same as exploration of the magnetic field parameters, examining a grid of points at $\pm 10°$ and $\pm 20°$ from the best point previously found. The process of recentring and recomputing the grid is repeated four times in total, to allow the programme to move to a set of parameters substantially different from the initially chosen set, if this proves necessary to obtain a reasonable fit to the data. At the end of four such surveys of parameter space, it is found in practice that the programme has generally identified a region of parameter space that is roughly consistent with the observations.

The programme then repeats the same search strategy four more times, but with parameter step sizes that are reduced to 0.4 of their previous values at each step. This results in a refined search for a best fit around the optimum set of parameters identified in the first cycle of four iterations. At the end of the refinement cycle, the best fit parameters are read out and the variations of B_ℓ, $B_{\rm s}$, etc. are computed with fine phase spacing for plotting and detailed comparison with the observations.

One subtlety of such a programme is the choice of weighting over the visible hemisphere of the various field components when they are calculated for comparison with observations. After some experimentation, the choice adopted was to assume a weighting by a product of limb darkening and line weakening of the form

$$W = [1 - \epsilon_c(1 - \cos\theta)][1 - \epsilon_l(1 - \cos\theta)]$$

where ϵ_c and ϵ_l are limb darkening and line weighting coefficients and θ is the usual angle between the local normal to the stellar surface and the line of sight. Because most of the data modelled here have been derived from polarized and unpolarized spectra centred around 6000 Å, these coefficients were determined by calculating continuum intensities and line depths for a sample Fe II line at 6150 Å, at various values of θ, for a series of effective temperatures. It was found that the two coefficients varied little enough over the range of effective temperature of the majority of stars studied (about 7500 to 10 000 K) that mean values of $\epsilon_c = 0.4$ and $\epsilon_l = 0.5$ could be used for this first exploration.

This method is very inefficient computationally, of course, but it is easy to programme and run. Using a PC with a 586 CPU and a Linux operating system, the programme runs in a few minutes, making it quite easy to try various experiments (such as starting from a variety of initial values of the model field parameters) to test if globally good solutions are found.

One of the ways in which the programme was tested was to try it on real data for several stars using specified starting values for the various parameters rather than the particular initial set described above. It was found that the programme was more likely to identify a globally optimum set of parameters if the initial value of the octupole component was set to $B_{\rm oct} = -0.25 B_{\rm d}$. The effect of the octupole component is to reduce or enhance the pole-to-equator contrast of the dipole, which affects the observed moments $B_{\rm s}$ and $B_{\rm mq}$ associated with a given B_ℓ; it appears that many observed fields are best fit by assuming that this contrast is less severe than for a pure dipole. On the other hand, the programme is quite successful at finding suitable values of $B_{\rm d}$ and $B_{\rm q}$ rather far from the initial choices.

It was found that in a number of cases, models with a fairly large range of values of $B_{\rm q}$ and $B_{\rm oct}$ fit the data about equally well. To encourage the programme to select small values of these field parameters when possible, a penalty function of the form $(B_{\rm q}^2 + B_{\rm oct}^2)/B_{\rm d}^2$ was added to the χ^2 computed from the fits to discourage unnecessarily large values of these higher multipole components.

The programme seems to be quite robust. It can be started from a variety of initial conditions and still converge on essentially the same final model, although if the initial conditions are chosen sufficiently poorly, it can fail to converge. In the few cases (e.g. HD 215441) in which the final field model was so far from the initial choices of the parameters that the programme could not find the best solution on a first try, the problem was obvious both from a large value of χ^2/ν and from the fact that some of the parameters were close to the limiting values that the programme could reach from the normal initial conditions. In each case, re-initializing the programme manually with new initial values and re-running it led to a satisfactory solution. It is found, in cases where the model cannot find a solution that fits all the data precisely, that the best model is somewhat sensitive to the choice of the errors (weights) assigned to the various field measurements, but this is not surprising, and represents a real ambiguity in the best choice of model. Individual cases of this sort are discussed in the next section.

3. Models of individual stars

The stars included in our sample are all the stars for which sufficient rotational phase coverage of at least B_ℓ and $B_{\rm s}$ is available that we know approximately the form of variation. In most cases observations of $B_{\rm mq}$ and $B_{\rm xover}$ are also available. Most of the published data may be found in Mathys et al. (1997), Mathys & Hubrig (1997), and the references cited there; a few data are taken from diverse sources such as Preston (1969), Huchra (1972), Borra & Landstreet (1978), and Hill et al. (1998); and a substantial number of the observations are still being prepared for publication by one of us (GM). The stars included in this study, along with some of their essential observational characteristics, and the parameters of the models found for each star, are listed in Table 1. The first two columns in this table list the HD number of each star and the rotation period P in days. The third column gives the value of the quantity $r = B_\ell^{\rm min}/B_\ell^{\rm max}$ where $B_\ell^{\rm min}$ and $B_\ell^{\rm max}$ are the extrema of variation of B_ℓ and the superscripts *min* and *max* refer respectively to the extrema of smaller and larger absolute value; the interest of this quantity will become clear below. The next five columns list the model fit values of the obliquity β of the field axis to the rotation axis, the inclination i of the rotation axis to the line of sight, and the polar field strengths $B_{\rm d}$, $B_{\rm q}$, and $B_{\rm oct}$ of the collinear dipole, quadrupole and octupole components of the field model. The final column contains remarks on the best model fit found.

Table 1 is divided into two parts on the basis of an important selection effect which is present for short-period stars but not for long-period ones. In order to be able to measure $B_{\rm s}$ in most magnetic Ap stars, the star must have $v \sin i$ less than roughly 6 km s^{-1}; otherwise, the rotational broadening of the line overwhelms the small Zeeman splitting. Now the equatorial velocity (in km s^{-1}) of a star of radius $R/R_\odot$ (in solar units) and rotational period P (in days) is

$$v_{\rm eq} = 50.6 (R/R_\odot)/P. \tag{1}$$

Thus, for an Ap star with a radius of order $R/R_\odot \approx 3$, $v_{\rm eq}$ is more than about 6 km s^{-1} for $P \le 25$ days. In this case, in order for $v \sin i$ to be less than 6 km s^{-1}, the star must be viewed from a favourable angle with respect to the rotation axis, and we expect that for such stars, in general $i \ll 90°$. In effect, in detecting $B_{\rm s}$ in stars of such short periods, we have made a very strong selection among all magnetic Ap stars for precisely those stars with small i. On the other hand, stars with periods of 25 days or more have such small values of $v_{\rm eq}$ (less than about 6 km s^{-1}) that we expect that among this group the distribution of i will be

Table 1. Magnetic field model parameters for stars studied

HD	P	r	β	i	$B_{\rm d}$	$B_{\rm q}$	$B_{\rm oct}$	Remarks
	d		°	°	G	G	G	
					Stars with $P < 25$ d			
65339	8.03	-0.92	86	50	-16700	-11200	5700	$i < \beta$
70331	1.99	0.93	69	5	-15300	26500	-3600	
137909	18.49	-0.60	85	15	-8700	-1400	-600	$i < \beta$; B_ℓ -$B_{\rm s}$ phase shift
142070	3.37	-0.27	83	9	7300	1100	-800	$B_{\rm mq}(calc) > B_{\rm mq}(obs)$
192678	6.42	1.0	32	4	4900	1300	2300	$i < \beta$
208217	8.44	-0.44	86	15	-13100	6000	-5000	
215441	9.49	0.53	30	30	62400	-42000	24200	
318107	9.71	0.26	78	11	23700	-23600	8300	
					Stars with $P > 25$ d			
2453	521.	0.45	11	62	-5000	-600	1800	
12288	34.9	0.07	22	62	-10100	-2800	4200	
14437	26.8	0.36	19	56	-10300	1600	6300	
51684	370.	0.50	18	42	-7300	-2200	2500	
61468	322.	0.38	19	49	-7400	-4100	-1000	$B_{\rm mq}(calc) > B_{\rm mq}(obs)$; B_ℓ -$B_{\rm s}$ phase shift ?
81009	33.96	0.64	11	48	7400	7200	2200	$B_{\rm mq}(calc) > B_{\rm mq}(obs)$
93507	556.	0.41	19	55	10900	-3700	-3400	$B_{\rm mq}(calc) > B_{\rm mq}(obs)$
94660	2700.	0.90	5	47	-8400	2700	6900	$B_{\rm mq}(calc) > B_{\rm mq}(obs)$
116114	27.6	0.92	2	56	-9000	1100	600	$B_{\rm mq}(calc) < B_{\rm mq}(obs)$
116458	148.4	0.68	10	52	-7600	2600	400	$B_{\rm mq}(calc) > B_{\rm mq}(obs)$
126515	129.95	-0.69	20	78	-13700	-17700	-5200	B_ℓ -$B_{\rm s}$ phase shift; asymmetric B_ℓ
144897	48.43	0.60	12	65	11000	-12900	-4900	$B_{\rm mq}(calc) > B_{\rm mq}(obs)$
166473	4400.	-1.00	35	87	-9400	-5700	1100	Period extrapolated from 2300 days of observations
187474	2345.	-1.00	45	86	-7700	-1600	1000	
188041	223.8	0.38:	10	70	5600	-1200	-1000	
200311	52.01	-0.98	24	88	12800	3800	800	asymmetric $B_{\rm s}$ curve

essentially random. No selection effect for any particular value of i is expected to be present in this sample.

Now recall that (as is well known) the values of β and i can be interchanged without altering the B_ℓ, $B_{\rm mq}$, or $B_{\rm s}$ curves predicted by the model, and thus we cannot distinguish one angle from the other by fits to the data used here. However, for the short-period stars HD 65339, 137909, and 192678 (Leroy et al. 1996; Wade et al. 1996) the values of i and β have been determined separately from linear polarization observations (which can clearly indicate which is the smaller angle). For all three stars the linear polarimetry indicates that $i < \beta$, which we have followed in our tabulation (although the angles in the table are from our own fits). These stars have been flagged with the remark "$i < \beta$". For the rest of the stars with $P < 25$ d, we have *assumed* that the smaller of the two angles found is generally i. For the long-period stars, where no selection effect on i occurs, we have made the opposite choice, and assigned the smaller of the two angles as β. The reason for this choice will be made clear in the next section.

Note that the values of χ^2/ν used by the programme to find a best fit do not have a precise meaning, as they depend on the nominal uncertainties assigned to each observed field moment as well as our estimates of the field moment values at the four nominal phases. In the initial fit the uncertainties are given values corresponding to our estimate of the actual accuracy of the data, but if the best model is not able to fit both $B_{\rm s}$ and $B_{\rm mq}$ simultaneously, the nominal uncertainty for $B_{\rm mq}$ is increased by a factor of two or so to give this quantity less weight relative to $B_{\rm s}$. Typically, the uncertainty assigned to the fitted values of B_ℓ is 50-200 G, that of $B_{\rm s}$ is 50-100 G, and that of $B_{\rm mq}$ is 500-1000 G (but for HD 215441 these uncertainties are several times larger).

The fits obtained for the stars for which we have reasonably complete data of our own are are shown in Fig. 1 through 5 for short-period stars, and in Fig. 8 through 17 for long-period stars. Although these figures demonstrate the overall good quality of the fits found, they also reveal limitations of the models and perhaps of the data.

Several kinds of mild disagreements between observed and calculated variations of field moments are observed, mostly in more than one star. Fig. 2 displays the field moments of HD 137909 = β CrB, for which the phase shifts between the extrema of B_ℓ and $B_{\rm s}$ clearly indicate that the field of the star is not exactly axisymmetric. Our axisymmetric model cannot fit the data in such a situation precisely, although it does furnish a reasonable first approximation. Similar indications of departures from axisymmetry are seen in small phase shifts or lack of reflection symmetry of magnetic curves around extrema in HD 208217

(Fig. 4) and perhaps HD 318107 (Fig. 5) among the short-period stars, and in HD 51684 (Fig. 7), HD 61468 (Fig. 8), and HD 126515 (Preston's star; Fig. 14) among the long-period ones.

Fig. 3 shows another difficulty of the fits for for HD 142070, for which complex low-level variation of $B_{\rm s}$ appears to be present around phase 0.90. This kind of fine structure in any field moment curve cannot be fit by a low-order model like ours. Most of our data for other stars do not have the extremely high signal-to-noise ratio needed to reveal such fine structure (the apparent asymmetric variations in the mean field modulus of HD 142070 have an amplitude of only a little more than 100 G), but a still smaller amplitude fine structure of this sort may be present in HD 208217 (Fig. 4) and HD 187474 (Fig. 17).

A third type of discrepancy occurs for several stars in which it is not possible to fit both $B_{\rm s}$ and $B_{\rm mq}$ simultaneously. Examples include HD 142070 (Fig. 3) among the short-period stars, and HD 61468 (Fig. 8), HD 81009 (Fig. 9), HD 93507 (Fig. 10), HD 94660 (Fig. 11), HD 116114 (Fig. 12) and HD 144897 (Fig. 15) among the long-period stars. In all but one case the discrepancy takes the form of a reasonably good model fit to $B_{\rm s}$ together with a calculated variation of $B_{\rm mq}$ which is similar in shape to the observed values, but 1 – 2 kG (10 – 20%) larger than the observed field (but in HD 116114 the computed $B_{\rm mq}$ is actually about 400 G smaller than the observed value). Note, incidentally, that in cases where the model does not seem to fit both $B_{\rm s}$ and $B_{\rm mq}$, we have forced the fit to agree with $B_{\rm s}$ by assigning to this field moment a much smaller uncertainty than that associated with $B_{\rm mq}$, in agreement with the relatively large σs obtained for the latter moment. In the case of this discrepancy, we suspect that part of the problem may lie somehow with a systematic effect in the observations of $B_{\rm mq}$, since in four of the stars in which this problem is found, the observed values of $B_{\rm mq}$ are about the *same* as the observed values of $B_{\rm s}$. This probably reflects some sort of problem, since (Mathys 1995b) the observed value of $B_{\rm s}$ is expected to measure $\langle B\rangle$, while $B_{\rm mq}$ is expected to measure $(\langle B^2\rangle + \langle B_{\rm z}^2\rangle)^{1/2}$, a quantity which should in general be significantly larger than $\langle B\rangle$. Thus we suspect that there may, at least in some cases, be a systematic error in the measured values of $B_{\rm mq}$.

The crossover field $B_{\rm xover}$ is clearly detected in three of the short-period stars: HD 137909, HD 142070, and HD 208217 (Fig. 2, 3, and 4), and perhaps in the long-period star HD 81009 (Fig. 9). In these cases a partial test of our models is possible, since our models predict a definite variation and phase relation for the $B_{\rm xover}$ data. However, the actual amplitude of the crossover field depends on $v \sin i$. Since we know both the period and (from our models) the value of i, we can estimate (from Eq. 1) the value of $v \sin i$. However, the derived value of $v \sin i$ depends on the actual radius of the star (which could be between about 1.75 and 3.5 $R_\odot$), and the calculated value of $B_{\rm xover}$ is also influenced somewhat by the assumed limb- and line-darkening coefficients assumed. Thus we cannot determine the amplitude of $B_{\rm xover}$ better than about plus or minus 50%. For all stars, the value of $v \sin i$ used to calculate the model variation of $B_{\rm xover}$ is determined from the stellar period and the model i value, assuming a value of $R/R_\odot = 2.5$.

It may be seen from the figures that almost all the undetected crossover fields are consistent with our models, and that the detected field of HD 137909 also agrees reasonably with our model computations, both in shape and phasing, and in amplitude. For HD 142070 and HD 208217 the phasing and shape are approximately correct, but the "normal" $v \sin i$ for these stars lead to amplitudes for the calculated $B_{\rm xover}$ curves that are larger than observed. That is, assuming that the magnetic models are correct, the assumed stellar radii are somewhat (30 - 50%) too large. In both cases, the deduction from this result would be that the stars are very close to the zero-age main sequence, which seems *a priori* somewhat unlikely. However, it seems equally plausible that the discrepancies are connected with the somewhat irregular shapes of the $B_{\rm s}$ curves, which hint at the presence of magnetic field distributions more complex than we can model with a simple multipole expansion. Alternatively, we might again have some unknown systematic effect in the measurements that leads to variation amplitudes that are somewhat too large.

Finally, note Fig. 17, which shows HD 187474, a star for which the variation of $B_{\rm s}$ is quite far from sinusoidal. We are able to reproduce this shape adequately with a model having a small quadrupole component – although we have not been able to find a model of HD 318107 (Fig. 5) which reproduces the flat part of its $B_{\rm s}$ curve. Nevertheless, we conclude that our model fits the data for all the stars at least approximately, apart from the modest discrepancies in the $B_{\rm mq}$ curves (which may not be due to shortcomings of the model) and inaccurate amplitudes for two or three $B_{\rm xover}$ curves. The fact that fits for the whole data ensemble are satisfactory suggests that the simple three-term collinear multipole expansion used provides a reasonably general first-order description of the surface geometries, although it is certainly not exact.

We next discuss the *uniqueness* of the fits. To start in a relatively general way, we note that we are primarily fitting models to the B_ℓ and $B_{\rm s}$ data. If these curves are sinusoidal, we have effectively four data values (say the extreme values of the sinusoidal variations), which we are fitting with a model that has 5 free parameters, i, β, $B_{\rm d}$, $B_{\rm q}$, and $B_{\rm o}$. Thus in many stars, we expect that the fit will not be able to provide a unique set of parameters. Only when we are able to use one of the other field moments as an effective constraint, or when we can use *shape* information about B_ℓ or $B_{\rm s}$ (as in HD 187474) can we expect the model to be uniquely defined. Since, in fact, many of the basic curves are nearly sinusoidal, we are clearly not able to define unique parameters for a number of stars, and so

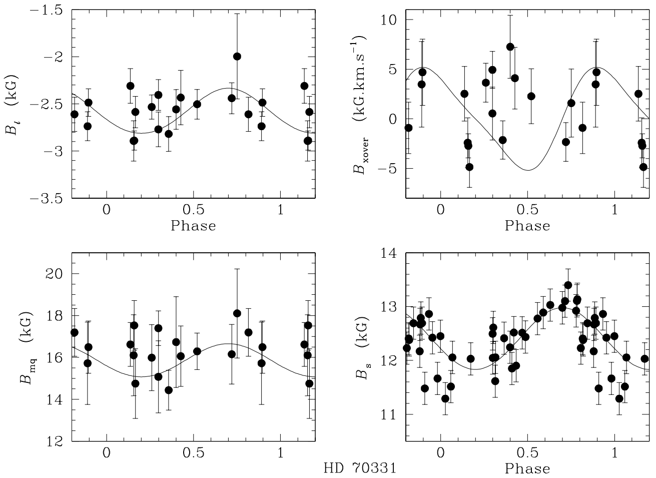

Fig. 1. Observations of HD 70331 compared to the best multipole model found, which has parameters given in Table 1. The four windows in the figure show (counter-clockwise from the top left) B_ℓ, $B_{\rm mq}$, $B_{\rm s}$, and $B_{\rm xover}$ as functions of rotational phase. Points with error bars are observations, and the smooth curves are the model fit.

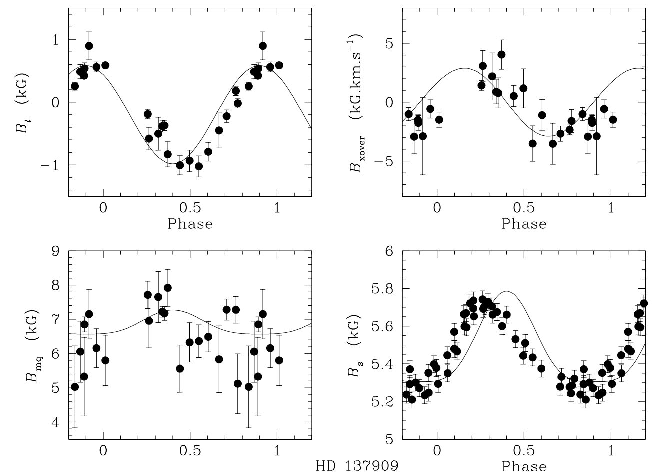

Fig. 2. Observations of HD 137909 = β CrB compared to the best multipole model found. Figure format as Fig. 1

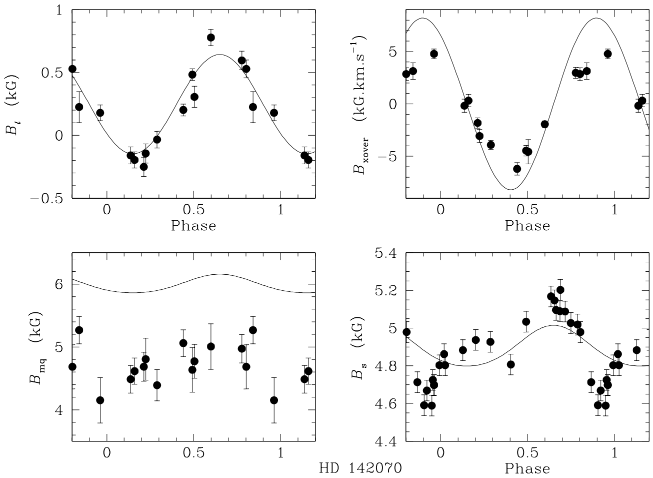

Fig. 3. Observations of HD 142070 compared to the best multipole model found. Figure format as Fig. 1.

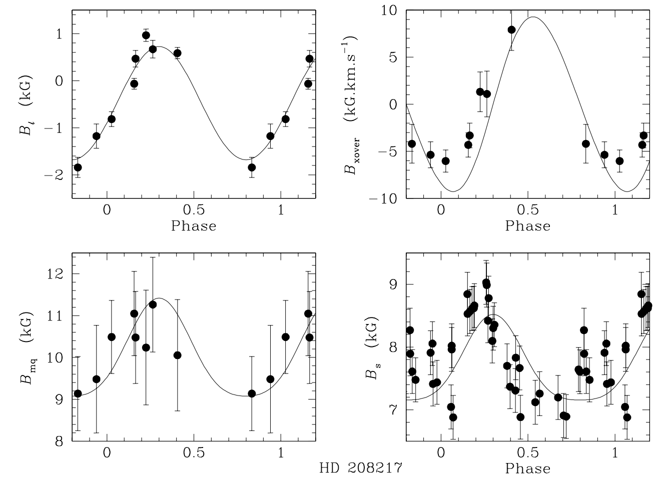

Fig. 4. Observations of HD 208217 compared to the best multipole model found. Figure format as Fig. 1.

we must be rather cautious about inferences drawn from values of the parameters given in Table 1.

At the start of this project, we hoped to be able to define fairly unique parameter sets for each stellar model by using shape information in the $B_{\rm s}$ curves. The reason that this is a realistic expectation is that each of the five model parameters has a distinctive effect on the computed variations of the various field moment curves. This is illustrated in Fig. 19, where we show the variations of B_ℓ (lower curves) and $B_{\rm s}$ (upper curves) for several values of the three field multipole parameters, as a function of the angle α between the magnetic axis and the line of sight to the star. (α is given by

$$\cos\alpha = \cos i \cos\beta + \sin i \sin\beta \cos 2\pi\phi, \qquad (2)$$

where ϕ is the rotational phase.) $B_{\rm mq}$ curves are not shown as they make the figure too crowded, but they closely follow the $B_{\rm s}$ curves with an offset of about 20% towards higher field values.

Four different models are included in this figure, all with $B_{\rm d}$ = 1000 G. They differ in the higher multipole values as described in the caption. These curves show the essential effects of the various multipole components used in these models. The variation of B_ℓ is almost exclusively defined by the value of $B_{\rm d}$. A non-zero value of $B_{\rm q}$ introduces a very small change in B_ℓ, but can substantially increase the value of $B_{\rm s}$ at one pole while decreasing it

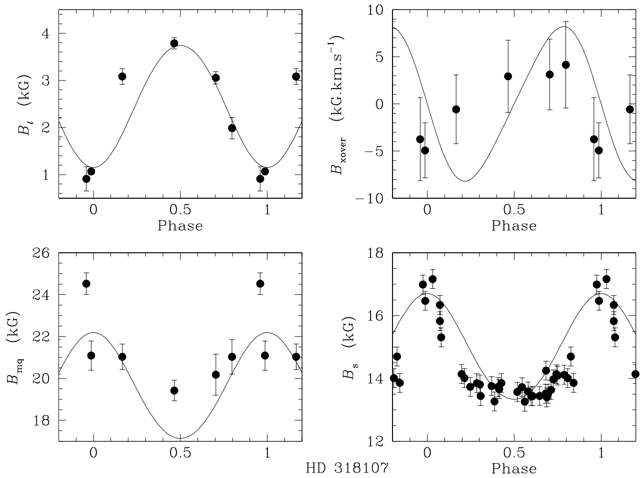

Fig. 5. Observations of HD 318107 compared to the best multipole model found. Figure format as Fig. 1

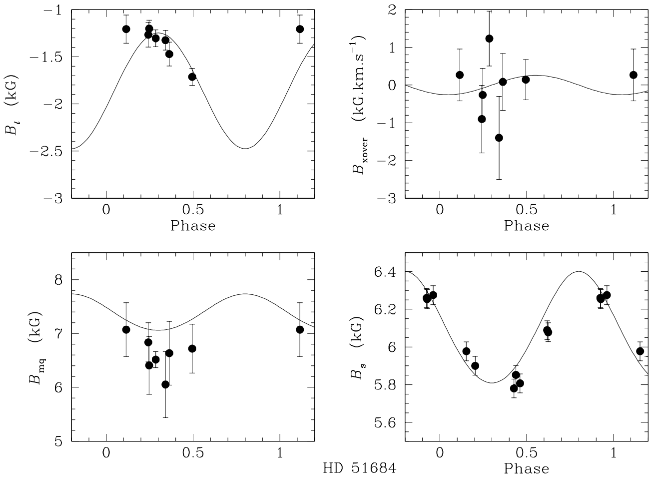

Fig. 7. Observations of HD 51684 compared to the best multipole model found. Figure format as Fig. 1.

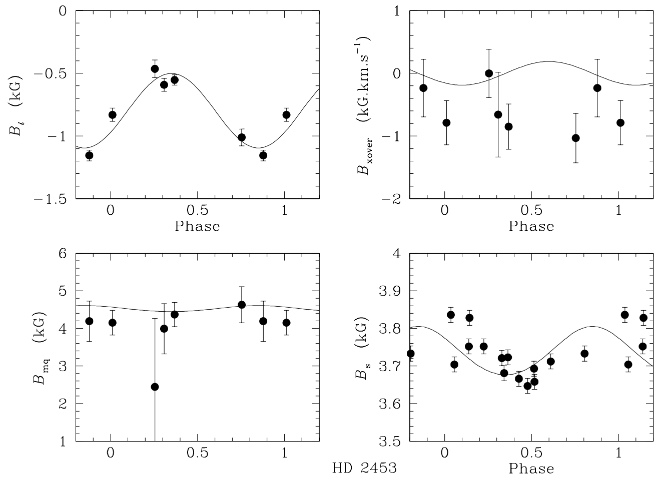

Fig. 6. Observations of HD 2453 compared to the best multipole model found. Figure format as Fig. 1.

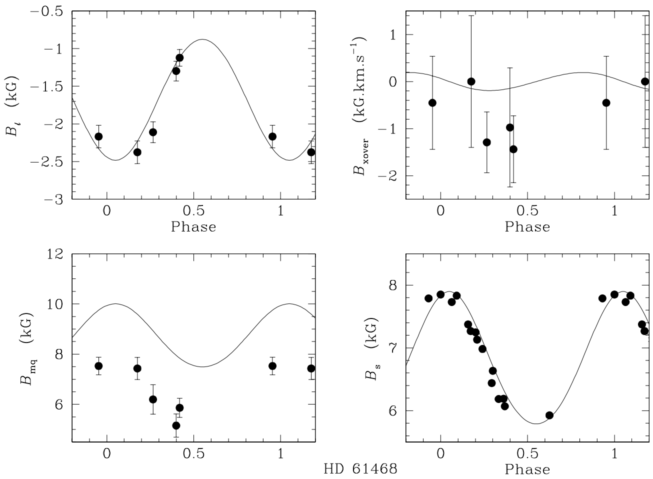

Fig. 8. Observations of HD 61468 compared to the best multipole model found. Figure format as Fig. 1.

at the other. A value of $B_{\rm oct}$ having the same sign as $B_{\rm d}$ increases the variation of $B_{\rm s}$ from the magnetic pole ($\alpha = 0°$) to the equator ($\alpha = 90°$), while the opposite sign reduces the pole-to-equator contrast. The added octupole field does not significantly change the variation of B_ℓ. Thus, we expect that for any star observed from well into one magnetic hemisphere to well into the other (i.e. for stars with large values of both i and β), the relative variations of B_ℓ and $B_{\rm s}$ should provide enough information to determine all three field moments with little ambiguity.

However, in the present sample of stars, we do not in general seem to have both i and β large. Instead, it appears from the models found that one of the angles (see Table 1) is almost always rather small, less than 30° in all but three models, and no more than 20° for three-quarters of the stars studied. We therefore do *not* generally sample a large fraction of the full pole-to-pole field moment variations. This has the consequence that the models derived are fit in most cases to only a relatively small fraction of the full pole-to-pole variation of the observational field moments, because the angle α between the line of sight and the magnetic pole ranges only between $\alpha = |(i-\beta)|$ and $\alpha = |(i+\beta)|$ (see Eq. 2). When one of β or i is small, these two angles are not very different, and so the field geometry is determined by observed moments arising from only a small part of the full variation possible if one could observe from $\alpha = 0$ to $\alpha = 180°$, as in Fig. 19. This may be visualized by imagining the magnetic axis nearly parallel to the rotation axis; as the star turns, the magnetic axis tilts a little closer to the observer, then a little further away, but never changes its inclination to the line of sight very much, so that the observed field moments

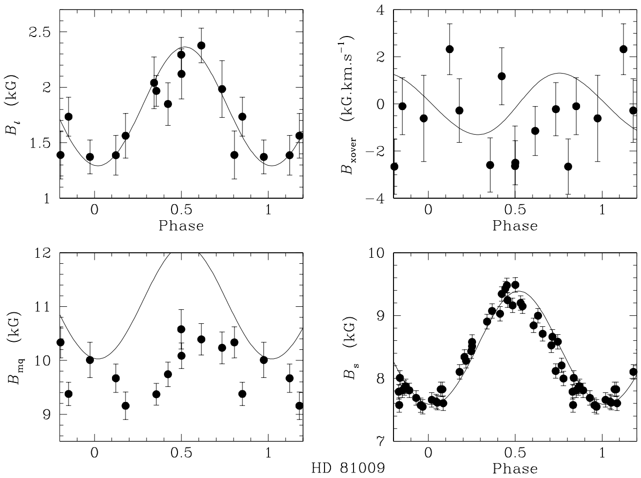

Fig. 9. Observations of HD 81009 compared to the best multipole model found. Figure format as Fig. 1.

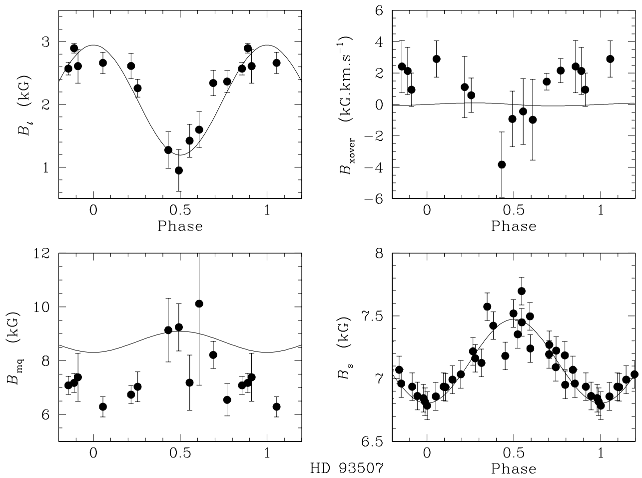

Fig. 10. Observations of HD 93507 compared to the best multipole model found. Figure format as Fig. 1.

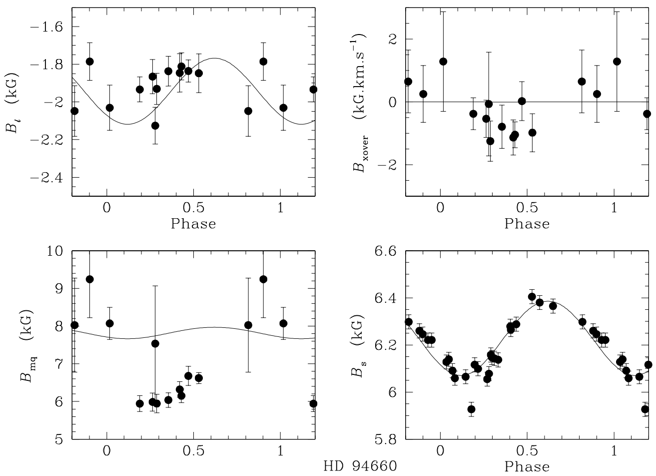

Fig. 11. Observations of HD 94660 compared to the best multipole model found. Figure format as Fig. 1.

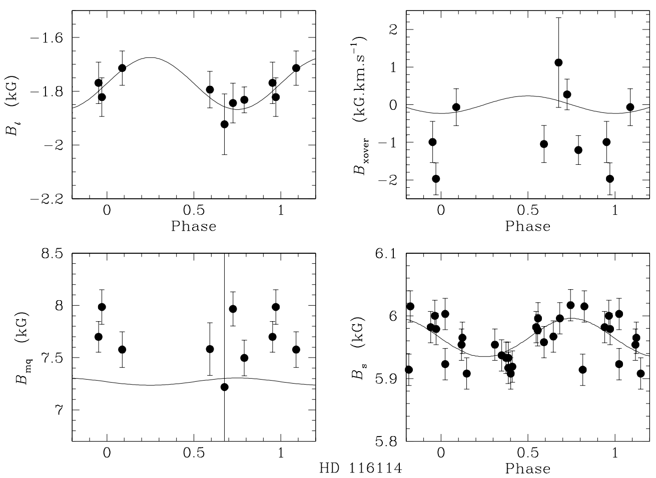

Fig. 12. Observations of HD 116114 compared to the best multipole model found. Figure format as Fig. 1.

are obtained from a rather limited range of views of the magnetic field.

The result is that for many of the stars discussed here, the values of the model parameters are not unique. This was clearly observed in the process of model fitting; quite different pairs of values of B_{q} and of B_{oct} can produce very similar fits to the observed data sets for a number of the stars. Even the larger of i and β, and the value of B_{d}, can be varied over substantial ranges and still produce good fits to the data, provided appropriate values of B_{q} or B_{oct} are chosen.

It is possible to understand how the various observed field moments behave when i or β is small. From Eq. 2, we see, as mentioned above, that a model of specified parameters has a "central" value of $\cos\alpha = \cos\beta\cos i$, and samples field moments from $\cos\alpha = \cos(i+\beta)$ to $\cos\alpha = \cos(i-\beta)$. Using this result in Fig. 19, it is clear that the presence of a small value of either i or β reveals itself either (a) when α is not near 90°, by a small amplitude of variation of B_ℓ (which does not change sign), or (b) if the central value of α is near 90°, by a (possibly reversing) B_ℓ combined with a relatively small ratio of B_ℓ/B_{s}. From examination of the stellar data shown in Fig. 1 through 18, we see that one of these two situations applies to *all* the stars shown except for HD 166473 and HD 187474. Thus it appears that the fact that the modelling programme found models with one small angle for most of the stars is a secure result, although the actual values of the smaller angle may be uncertain by a few degrees, and the rest of the parameters are quite non-unique.

Are any of the other parameter determinations secure? In fact, another well-determined aspect of the model pa-

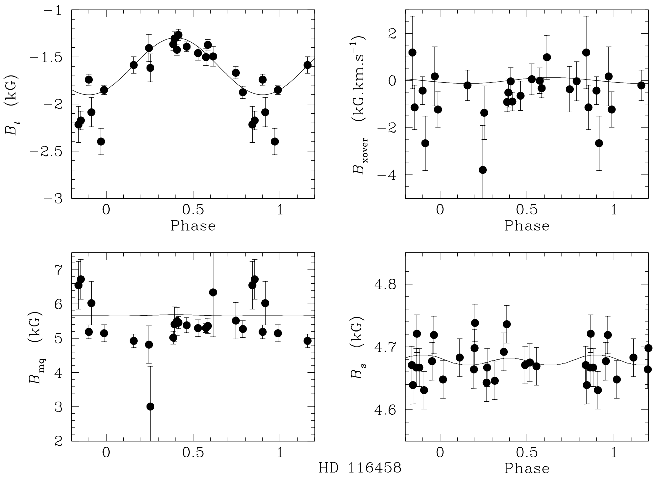

Fig. 13. Observations of HD 116458 compared to the best multipole model found. Figure format as Fig. 1

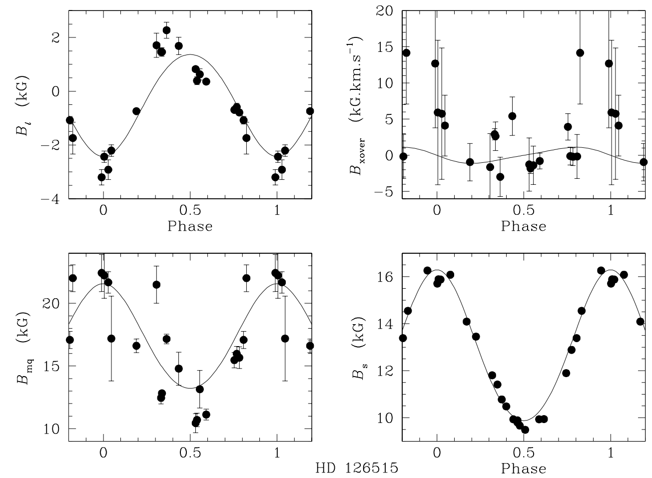

Fig. 14. Observations of HD 126515 compared to the best multipole model found. Figure format as Fig. 1

rameters is the occurrence of a non-zero value of either $B_{\rm q}$ or $B_{\rm oct}$ for a number of stars. From Fig. 19, it can be seen that for a pure dipole model, as $|B_\ell|$ increases from zero (with either sign), $B_{\rm s}$ also increases. $B_{\rm s}$ can only *decrease* as $|B_\ell|$ increases from zero by the addition of a non-zero $B_{\rm q}$ or $B_{\rm oct}$. However, a number of the stars do show $B_{\rm s}$ values that decrease as $|B_\ell|$ increases, at least over a part of their cycle. Examples include HD 70331, HD 208217, HD 318107, HD 2453, HD 93507, HD 126515, HD 144897, HD 166473, and HD 187474. For these stars at least, the requirement for a non-zero value for $B_{\rm q}$ or $B_{\rm oct}$ is clearly established, although the actual value is not.

Thus, one of the principal goals of this study, namely to define reasonably unique multipole models for the stars in this sample, has proven not to be possible, because almost all of the stars in the sample have either i or β small. However, although most of the parameter values derived by the modelling process for the stars of the current sample must be regarded as non-unique (often several substantially different parameter sets fit the observations about equally well), it is nevertheless clearly established that almost all of the stars observed *do* have one of i or β rather small, and a number of the stars require a substantial quadrupole or octupole component to fit the observations.

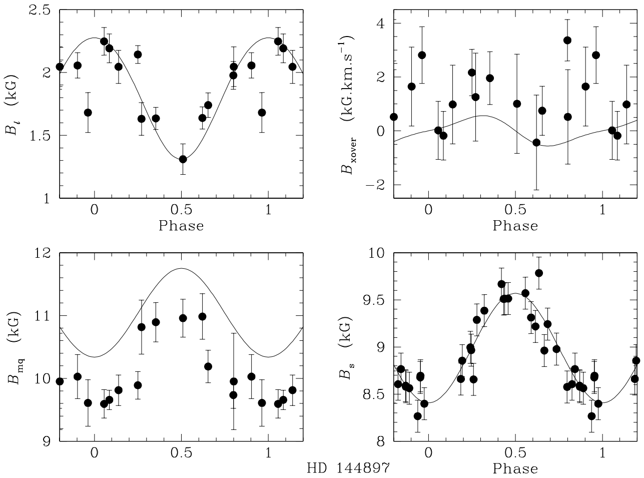

Fig. 15. Observations of HD 144897 compared to the best multipole model found. Figure format as Fig. 1

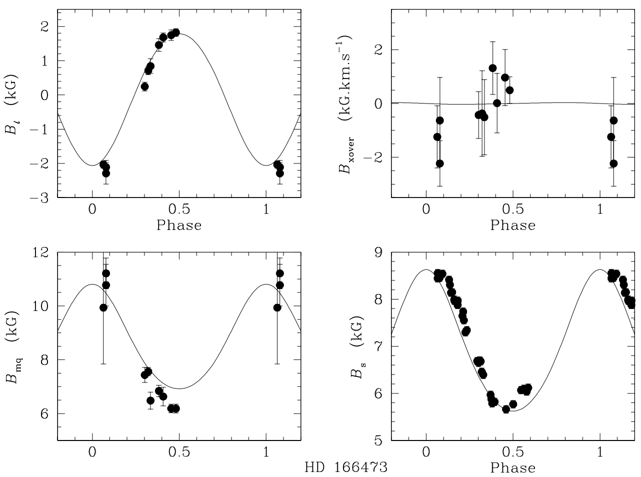

Fig. 16. Observations of HD 166473 compared to the best multipole model found. Figure format as Fig. 1.

4. Properties of the ensemble

We now turn to an examination of the regularities seen in the derived field structures of this large sample of magnetic stars.

When we examine the parameters in Table 1, as discussed above, we see that one of the two angles is almost

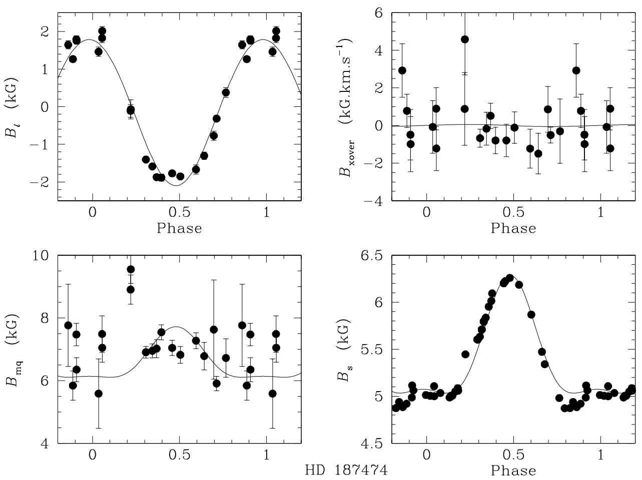

Fig. 17. Observations of HD 187474 compared to the best multipole model found. Figure format as Fig. 1

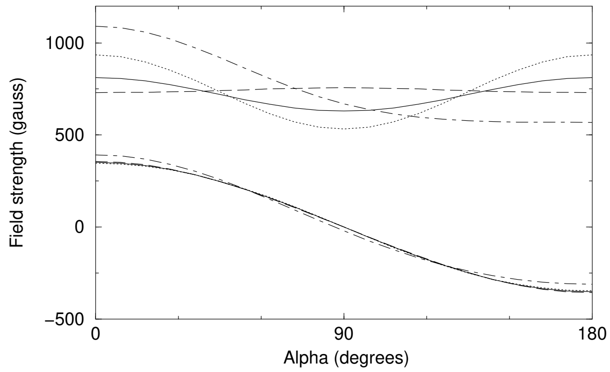

Fig. 19. Variations of B_ℓ (lower curves) and $B_{\rm s}$ (upper curves) with angle α from one magnetic pole to the other for several different field models, all with $B_{\rm d} = 1000$ G. The solid curves show B_ℓ and $B_{\rm s}$ for both $B_{\rm q}$ and $B_{\rm oct}$ equal to 0; the dot-dash curves are for $B_{\rm q} = 500$ G and $B_{\rm oct} = 0$; and the dotted and long-dash curves are for $B_{\rm q} = 0$ and $B_{\rm oct} = +500$ and –500 G respectively.

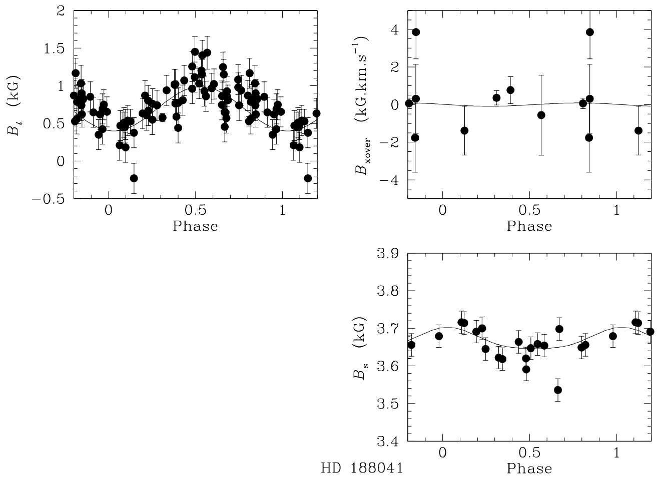

Fig. 18. Observations of HD 188041 compared to the best multipole model found. Figure format as Fig. 1. The mean quadratic field of this star could not be determined from the observations.

always less than 30°, both for the short-period and the long-period stars. This result has emerged robustly from use of our modelling programme even as various combinations of the remaining parameters were found that fit the observations. We believe that – in spite of the fact that most model parameters are poorly determined for these stars – the modelling result that one of i and β must be small for almost all the stars in the sample is well-established.

For the short-period ($P \leq 25$ d) stars, the smallness of the angle i is expected, as we have discussed above, since it is necessary for surface field measurements to be possible. On the other hand, the systematic occurrence of a small angle for the long-period stars is completely unexpected. We cannot determine from the present data whether it is β or i which is small. However, for the long-period stars, because of the low values of $v_{\rm eq}$, we have no selection effect enhancing the number of stars of small $\sin i$, and hence we expect that the rotation axes must have random orientations in space. This is equivalent to the statement that the visible rotation pole is equally likely to be anywhere on the visible hemisphere of the star. This leads directly to the conclusion that small values of i are rather unlikely to occur, since they require the visible rotation pole to be within a small region near the centre of the visible hemisphere. The probability in a large ensemble of stars with randomly orientated axes that i will be less than some value i_0 is given by

$$P(i < i_0) = 1 - \cos i_0.$$

The probability of having $i < 30°$ is only about 13.4%. Among the 16 stars of Table 1 with $P > 25$ days, probably approximately two have $i < 30°$.

To demonstrate that the assumption that the larger of the two model angles in the long-period stars is generally i is consistent with the expected distribution of i values, we show in Fig. 20 the cumulative distribution of model i values from Table 1, where it is assumed that i is *always* the larger of the two angles, compared to the theoretical distribution. In general, the agreement between the two curves is satisfactory. The largest difference vertical between the two curves, a difference of 0.26, is only significant (using a two-sided Kolmogorov test) at about the 80% confidence level. Even this marginal difference is presumably due to the fact that we have chosen to make i the larger of the two angles in every case, although there are about two stars for which this assumption is presumably incorrect.

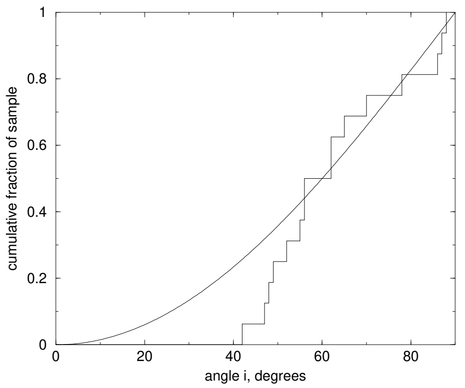

Fig. 20. The cumulative distribution of values of inclination i expected theoretically for stars with randomly distributed rotation axes (smooth curve) is compared to the distribution of i values deduced from modelling for the long-period stars, assuming that i is always the larger of the two angles found.

However, 14 of the 16 long-period stars have *one of* i or β less than 30°. Since this small angle is most probably not i for 12 of these stars, the small angle must be β in almost every case. This is the reason that we have chosen to tabulate β as the smaller of the two angles found by model fitting in Table 1.

It is clear that somewhat larger values of β *can* occur in long-period stars: in our sample, HD 166473 and HD 187474 both appear to have $30° \leq \beta \leq 45°$, assuming that the smaller angle is β. However, it is remarkable that not a single long-period star is found to require both angles (and thus *a fortiori* β) larger than 45° for a reasonable model, although this condition was *not* imposed in the modelling process.

There is not a large body of modelling for the short-period stars with which to compare this result, partly because $B_{\rm s}$ cannot be measured in the rotationally broadened lines of most short-period stars, leading to large uncertainties in the deduced values of both i and β, and partly because modelling based only on intensity and circularly polarized spectra does not usually allow one to distinguish between i and β. We argue for most of the stars in Table 1 (on the basis of small values of $v \sin i$) that β is very probably the larger angle; here we find two stars (out of eight in the table) for which β is about 30°, while for the other six short-period stars β is not far from 90°. Another dataset in which i and β can be distinguished, and in which both angles are determined with reasonable precision, is in the analysis by Leroy et al. (1996) of Leroy's broadband linear polarization observations. Of the 13 short-period Ap stars modelled there, only three (HD 24712, HD 80316, and HD 152107) have β smaller than 45°, and none has β less than 30°. It thus appears that the predominance of small values of β (often smaller than 20°) among the long-period stars is rather different from the situation prevailing among the short-period stars, where β may sometimes be as small as about 30° but is normally considerably larger.

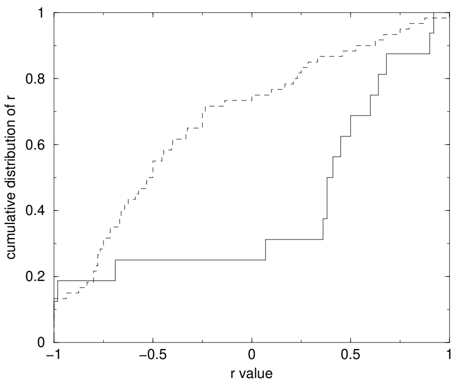

Fig. 21. The cumulative distribution of r values observed in the present sample of long-period stars (solid curve) compared to the same distribution for 60 short-period stars of well-determined r values found in the literature. It is clear that about 75% of the short-period stars have fields that reverse sign, while about 75% of the long-period stars have fields that do not reverse. The lack of field reversals in the long-period stars implies that they have systematically small values of β.

This surprising result is consistent with another unexpected characteristic of the sample of long-period stars. About 75% of these stars have measured variations of B_ℓ which *do not* change sign during the stellar rotation, and thus have $r > 0$. This is radically different from the distribution of this quantity for samples of more rapidly rotating stars, where about 75% of the stars have $r < 0$. In Fig. 21 we compare the cumulative distribution of r values observed in the present sample of long-period stars with the distribution observed in a sample of about 60 stars of known (short) periods and well-determined r values which have been collected from the literature. The distribution of r values for the short-period stars, although more extensive than previously reported distributions, is consistent with them (e.g. Landstreet 1970). The figure clearly shows the great difference between the two distributions. With a largest difference between the two distributions of about 0.56, the two distributions are different (according to the two-sided Smirnov test) at more than the 99% confidence level.

Preston (1967) and Landstreet (1970) have shown that the distribution of r is a good indicator of the distribution of β angles, and that the predominance of $r < 0$ in bright magnetic Ap stars indicates that most such stars have large values of β, of the order of 60° or more (see also Hensberge et al. 1979). The actual distribution of β values for short-period stars is not well determined, but the observed r distribution was found by the authors cited above to be consistent with a random distribution of orientations of the magnetic axis relative to the rotation axis.

That is, the distribution of observed r values is consistent with the assumption that in a sample of stars, the magnetic axis of the star can emerge anywhere on the stellar surface. A random distribution of magnetic axes relative to the position of the rotation axis on the star leads to predominantly large values of β, again because small values of β require the magnetic axis to lie within a small region on the surface of the star near the rotation axis. However, in the present sample of long-period stars, the predominance of *positive* values of r indicates that the opposite situation occurs, namely that β is generally *small*, typically of the order of 20 or 30° (Landstreet 1970).

We are thus led to the completely unexpected conclusion that, unlike the short-period magnetic Ap stars, most magnetic Ap stars with periods longer than about 25 days have small values of β; that is, their magnetic axes have a strong tendency to be rather closely aligned with their rotation axes.

5. Discussion and conclusions

In modelling the greatly enlarged sample of magnetic Ap stars for which enough kinds of data (particularly *both* B_ℓ and B_s) are available to make it possible to determine several parameters for a simple model of the magnetic field geometry, we have found the following results.

First, the simple model of a low-order axisymmetric multipole field expansion of three terms (co-axial dipole, quadrupole and octupole) is found to be able to reproduce approximately the mutual variations of the observed field moments B_ℓ, B_s, B_mq and B_xover in all the stars in our sample. It appears that this model is an adequate first approximation to the field structure of all the stars of the present sample, and by extension, to the field structures of other magnetic Ap stars.

Secondly, when we fit such models to the observed field moments, we find that significant quadrupole and octupole terms are often required. These terms are sometimes even of the same magnitude as the dipole term. However, most of the parameters in Table 1 are not determined uniquely because the range of variations of the observed field moments is often surprisingly small.

Finally, we have found that the observations of stars of $P > 25$ days indicate that most of these stars have small values of the obliquity β of the model magnetic axis to the rotation axis, of the order of 20°. This result is completely unexpected, as it is the opposite of the situation for the commoner rapidly rotating magnetic Ap stars, most of which have β values of 60° or more. This surprising feature of the models provides an explanation for the fact that relatively few of the long-period stars have B_ℓ fields that change sign during the stellar rotation period. (The fact that half of the present sample of short-period stars do not show reversing B_ℓ fields is because this sample is strongly selected for small values of i.)

This final conclusion, that among the stars of $P > 25$ days the value of the obliquity β is usually small, constitutes a fundamentally new fact about the magnetic fields of magnetic Ap stars. This result implies that the process of losing angular momentum is somehow linked to a process that determines the field obliquity, or that variations in angular momentum from one star to another have important effects on the later evolution of the surface field structure. In any case, the observed correlation between long rotation period and small obliquity provides an important clue bearing on both the problem of how magnetic Ap stars lose angular momentum and on the problem of how the observed field structure is produced.

We therefore examine some of the mechanisms which may affect the obliquity β or which may lead to the very long rotation periods that we find to be correlated with small values of β. We start with mechanisms that may operate during the main sequence lifetime of the star (where we have some definite observational constraints) and then consider the pre-main sequence period.

Possible effects leading to systematic relationships between the magnetic and rotation axes have been discussed in theoretical studies of the dynamical effects of distortion of the figure of the star by the magnetic field, in studies of the structure and evolution of the internal magnetic field of the Ap stars, and in studies of the effects of mass loss or accretion on angular momentum and field structure.

In one class of effects, the star has no interaction with the outside world but changes obliquity because of internal motions. Mestel & Takhar (1972) have shown that if the internal magnetic field of a magnetic Ap is sufficiently large to produce significant distortion of the figure of the star, then nutational motion occurs that is similar to that arising in a top rotating about an axis other than a principal axis of inertia. In this case dissipation may lead to alignment or orthogonality of the two axes in an interestingly short time. The obliquity β will tend to zero if the overall figure of the star is oblate about the magnetic axis (presumably due to a predominantly poloidal internal magnetic structure), while a prolate figure (which could be produced by a dominant internal toroidal field) should lead eventually to a large value for β. Alignment might be produced either during the pre-main sequence phase or on the main sequence. This effect could be involved in the correlation we find between slow rotation and small obliquity if the internal field structure in a star is strongly affected by its rotation. What would be required is that in slow rotators the field structure would need to be predominantly poloidal, while inside more rapid rotators differential rotation would produce a predominantly toroidal field structure. We might imagine that most magnetic Ap stars reach the main sequence with small obliquity (say of order 30° or less) but a range of rotation rates; the more rapidly rotating stars could develop important toroidal field structures through differential rotation, and then be driven by dissipation of nutational internal motions to a large obliquity. If this hypothesis is the correct mechanism by which the obliquity of slow and rapid rotators becomes different, it suggests that there should be some important population of young rapidly rotating stars with small β values, of a size that depends strongly on the time scale for change of β. Although there are almost certainly some rapid rotators with small obliquity, there may well not be as many as required by this hypothesis, and it is not currently known whether such stars are systematically young.

Another mechanism which probably changes the obliquity of the stellar field on an evolutionary time-scale without interaction with the external world is the field advection produced by internal "meridional" circulation currents (Eddington-Sweet circulation) together with ohmic dissipation. The effects of this mechanism are discussed by Moss in a series of papers (Moss 1984, 1985, 1987, and 1990). Moss shows that the effect of internal circulation currents is to increase β with time; in fact, if stars arrive on the main sequence with magnetic axes randomly distributed over the stellar hemisphere relative to the rotation axis, Moss (1990) predicts that an even larger fraction of all magnetic Ap stars should have $r \approx -1$ than is observed. He suggests that the magnetic Ap stars may arrive on the main sequence with initial values of β which are usually small compared to 90°, and that the value of β then increases with time because of field advection. In this case, the slowest rotators would have the slowest circulation currents, and would thus tend to retain their initial small β values, in agreement with our observations. Thus this mechanism too may play a role in the correlation we have discovered.

Magnetic braking by a stellar wind, which can change both the rotation rate and the magnetic obliquity, has been discussed in several papers culminating in Mestel & Selley (1970). These authors argue that the braking torque produced by mass loss in the presence of a magnetic field with β different from 0° or 90° would cause the axis of rotation to precess through the star, altering β in the process. The precession shuts off when β reaches either 0 or 90°. The calculations indicate that which limit is reached depends on the detailed surface field configuration, with the strong-field regions tending to move towards the rotational poles. In the case of a dipole-like field, this could reasonably lead to field alignment. For this mechanism to be relevant to the correlation we have found, we might postulate a stellar wind whose duration varies greatly (for some reason) from star to star. In this case, the stars for which the wind persists longest, and which are therefore most strongly slowed down, would also tend to end with their fields (assuming that these fields have the correct surface structure) aligned with the rotation axis.

The related process of angular momentum loss and obliquity change via magnetized accretion has been discussed by Mestel (1999). As in the case of magnetic mass loss, the stars most strongly braked would probably also suffer the greatest change to the obliquity angle, possibly leading to alignment. However, this mechanism suffers from the added uncertainty as to whether accretion would in fact brake the star or not – accretion can certainly add angular momentum to a star as well as removing it.

Mechanisms slowing the rotation of magnetic stars, and perhaps changing or influencing the obliquity, may also operate during the pre-main sequence phase. One or more of the mechanisms discussed above could operate during the pre-main sequence period. Stępień (2000) has recently argued that the rotation period of most magnetic Ap stars is determined by a combination of interaction with and accretion from a disk, mass loss via a wind, and change of moment of inertia. In his model, the most slowly rotating magnetic Ap stars are stars with exceptionally large magnetic fields which, while still pre-main sequence stars, have (for some reason) continued to lose mass via a magnetized wind long after interaction with a disk ends. The pre-main sequence stage is long enough (at least for the lower mass Ap stars which are observed to have very long periods) that angular momentum loss via a wind could produce the long periods observed. If Stępień's view is correct, several possible explanations for our correlation are apparent. First, one could imagine that stellar rotation is slowed during the pre-main sequence phase, followed later by the development of large β values in rapid rotators, but not in the most slowly rotating stars, by field line advection (Moss 1990). Another possibility is that pre-main sequence angular momentum loss could lead to different internal field structures, poloidal in the slowest rotators but toroidal in the more rapidly rotating stars, with consequent evolution towards small or large β values respectively because of nutation (Mestel & Takhar 1972). Yet a third possibility is that the excessive angular momentum loss that produces the slowest rotators would also tend to lead at the same time to small obliquity via the torque exerted on the field (Mestel & Selley 1970). Finally, perhaps the long duration of a wind postulated by Stępień as the cause of excessively slow rotation is in some way related to surface magnetic structure, with stars with fields of small obliquity having winds for a longer time than stars of larger β. However, it is not obvious how this result might be produced physically.

Thus, we have a number of possible explanations of the correlation that we observe between unusually slow rotation and aligned magnetic and rotation axes. Further theoretical study and modelling will be necessary to decide which – if any – of the possibilities discussed above is correct. In any case, the observed correlation provides one important new constraint that should be of value in such theoretical studies.

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References

Babcock, H. W., 1947, ApJ 105, 105
Babcock H. W., 1960, ApJ 132, 521
Bagnulo, S., Landolfi, M. and Landi Degl'Innocenti, M., 1999, A&A 343, 865
Bohlender, D. A., Landstreet, J. D., and Thompson, I. B., 1993, A&A 269, 355
Borra, E. F. and Landstreet, J. D., 1978, ApJ 222, 226
Didelon, P., 1983, A&AS 53, 119
Hensberge, H., van Rensbergen, W., Goossens, M., and Deridder, G. 1979, A&A 75, 83
Hill, G. M., Bohlender, D. A., Landstreet, J. D. et al., 1998, MNRAS 297, 236
Huchra, J. 1972, ApJ 174, 435
Landolfi, M., Landi Degl'Innocenti E., Landi Degl'Innocenti M., and Leroy, J. L., 1993, A&A 272, 285
Landstreet, J. D., 1970, ApJ 159, 1001
Landstreet, J. D., 1980, AJ 85, 611
Landstreet, J. D., 1982, ApJ 258, 639
Landstreet, J. D., 1988, ApJ 326, 967
Landstreet, J. D., 1990, ApJ 352, L5
Landstreet, J. D., Barker, P. K., Bohlender, D. A. and Jewison, M. S., 1989, ApJ 344, 876
Leroy, J.-L., Landolfi, M. and Landi Degl'Innocenti, E., 1996, A&A 311, 513
Mathys, G., 1989, Fund. Cos. Phys., 13, 143
Mathys, G., 1990, A&A 232, 151
Mathys, G., 1995, A&A 293, 733
Mathys, G., 1995, A&A 293, 746
Mathys, G. and Hubrig, S., 1997, A&AS 124, 475
Mathys G., Hubrig, S., Landstreet, J. D. et al., 1997, A&AS 123, 353
Mestel, L. 1999, *Stellar Magnetism* (Oxford: Oxford University Press), ch. 8.
Mestel, L. and Selley, C. S. 1970, MNRAS 149, 197
Mestel, L. and Takhar, H. S. 1972, MNRAS 156, 419
Moss, D. 1984, MNRAS 209, 607
Moss, D. 1985, MNRAS 213, 575
Moss, D. 1987, MNRAS 226, 281
Moss, D. 1990, MNRAS 244, 272
Preston, G. W. 1967, ApJ 150, 547
Preston, G. W., 1969, ApJ 156, 967
Semel, M., 1989, A&A 225, 456
Stępień, K. 2000, A&A 353, 227
Wade, G. A., Elkin, V. G., Landstreet, J. D. et al., 1996, A&A 313, 209
Wade, G. A., Donati, J.-F., Landstreet, J. D. and Shorlin, S., 2000, in preparation