Elementary particles in nuclear physics

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1 Introduction

- Up to now we have been dealing with a mostly exact and complete description of the forces (electromagnetic) and particles (electrons and "structure-less" nuclei) relevant to our problems.
- Now we turn to the study of nuclei: here the basic particles do have underlying structure that is fundamental to their interactions, especially through the nuclear (strong) force.
- This underlying structure is (a) incompletely understood and (b) quite complicated, so we will develop as complete an understanding of nuclei as we can, using familiar wave mechanical models, *without* examining the underlying structure ("elementary particle theory") in detail.
- Nevertheless, it is worthwhile to briefly introduce the vocabulary and a few results from this theory before starting on nuclei themselves.

1.1 Fermions, bosons, and statistics

- All particles are either fermions (spin 1/2, 3/2, etc), like electrons, or bosons (spin 0, 1, 2, etc) like photons.
- Fermions have anti-symmetric wave functions under exchange of two identical particles, and obey exclusion the principle. Bosons have symmetric wave functions under exchange, and do not obey the exclusion principle.

1.2 The fundamental interactions

- Particle physics identifies four fundamental interactions among fermions. The interactions are said to occur through fields, and the particles associated with these fields are bosons. Thus we have

  1. the strong interaction, carried by gluons ($s = 1$)
  2. the electromagnetic interaction, carried by photons ($s = 1$)

3. the weak interaction, carried by W and Z intermediate bosons ($s = 1$)
4. the gravitational interaction, carried by gravitons ($s = 2$)

- *Quarks* seem to be the basic constituents of the fermions which interact via the strong interaction (protons, neutrons, etc). Strongly interacting particles are called *hadrons*.

1.3 Conservation laws

- Conservation laws play an extremely important role in our understanding of all kinds of physical systems. For elementary particles, some are familiar and some are not.
- Familiar conservations laws include conservation of energy, and of linear and angular momentum.
- We have already seen that parity is an important property of atomic an molecular systems. A state has parity if

  $$\psi(-\mathbf{r}_1, -\mathbf{r}_2, \ldots) = +\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots) \quad \text{or} \quad -\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots).$$

  It is found that as a system evolves under the action of the strong and electromagnetic forces, it conserves parity. This is not true for evolution under the weak force.

2 Leptons, photons and their interactions

2.1 The electromagnetic interaction; photons

We may motivate the qualitative picture of elementary particles by recalling some characteristics of light.

- The EM field satisfies wave equations for $\mathbf{A}$ and ϕ:

  $$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho(\mathbf{r}, t)}{\varepsilon_0}.$$

  Where $\rho = 0$, this equation has propagating wave solutions

  $$\phi(\mathbf{r}, t) = C \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$$

  where the wave parameters $\mathbf{k}$ and ω must satisfy the dispersion relation $\omega^2 = c^2k^2$. If we apply the energy and momentum operators

  $$i\hbar\frac{\partial}{\partial t} \quad \text{and} \quad -i\hbar\nabla$$

to this wave to determine the characteristic energy and momentum, and then use the dispersion relation, we find $E^2 = p^2c^2$, a special case of Einstein's relation

$$E^2 = p^2c^2 + m^2c^4$$

which suggests that the quantum of EM energy has no rest mass.

- Another important kind of solution occurs when time variations may be neglected. From a point charge Q, the solution is

$$\phi(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}.$$

- Scattering may be viewed as due to the exchange of a *virtual* photon which transfers momentum from one charged particle to another. Such virtual particles are a major aspect of elementary particle theory; they are unobservable particles which come into existence only for the time allowed by the uncertainty principle, so their existence does not violate conservation of energy. They may be exchanged between real (observable) particles, carrying momentum, energy, charge, etc as long as no conservation laws are violated. These virtual particles are the carriers of the interaction fields.

2.2 The weak interaction; intermediate bosons

Physicists now describe the weak interaction as being carried by three weak interaction fields associated with three intermediate bosons, the W^+, W^- and (neutral) Z. Each is described by a vector and a scalar potential, like the EM field. These bosons have masses of the order of 100 m_p.

- If we guess that the generalization of the EM wave equation for the scalar field (there is also a vector field) carried by a massive Z particle is

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \left(\frac{M_\mathrm{Z}c}{\hbar}\right)^2\right]\phi_\mathrm{Z}(\mathbf{r}, t) = -\frac{\rho_\mathrm{Z}(\mathbf{r}, t)}{\varepsilon_0},$$

then again where the weak charge $\rho_\mathrm{Z} = 0$ there are plane wave solutions

$$\phi_\mathrm{Z}(\mathbf{r}, t) = C\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$$

but the dispersion relation is $\omega^2 = c^2k^2 + c^2(M_\mathrm{Z}c/\hbar)^2$ in agreement with the Einstein mass-energy relationship.

- The solution for a point weak charge Q_Z (N.B.: not an EM charge) is

$$\phi_\mathrm{Z}(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q_\mathrm{Z}\mathrm{e}^{-\kappa r}}{r}, \quad \text{where} \quad \kappa = \frac{M_\mathrm{Z}c}{\hbar}.$$

The value of κ^{-1} for the intermediate bosons is about 2×10^{-3} fermi (1 fermi is 10^{-15} m), *much* smaller than the size of a nucleus. Beyond a few times κ^{-1} the potential is constant at 0.

- Generalizing to a charge distribution, the weak potential is

$$\phi_{\mathrm{Z}}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_{\mathrm{Z}}(\mathbf{r}',t)\mathrm{e}^{-\kappa|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}^3\mathbf{r} \approx \frac{1}{4\pi\varepsilon_0}\rho_{\mathrm{Z}}(\mathbf{r},t)\int \frac{\mathrm{e}^{-\kappa|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}^3\mathbf{r}'$$

$$= \frac{1}{\varepsilon_0}\left(\frac{\hbar}{M_{\mathrm{Z}}c}\right)^2 \rho_{\mathrm{Z}}(\mathbf{r},t).$$

  The potential is *zero* unless the two particles are in contact.

- The short range of the weak force can be understood using the idea of virtual particles that come into existence only to the extent they are allowed by the uncertainty principle. From $\Delta E \Delta t \geq \hbar$ we see that a particle of mass energy $M_{\mathrm{Z}}c^2$ could exist – without violating conservation of energy – for a time of order $\Delta t \sim (\hbar/M_{\mathrm{Z}}c^2)$, during which time it could move from its origin to a distance of order $\ell \sim c\Delta t \sim (\hbar/M_{\mathrm{Z}}c) = \kappa^{-1}$. Because the weak force is carried by massive bosons, its range is limited to the region which can be reached by such heavy virtual particles. In contrast, since there is no minimum energy that a photon may have, low energy virtual photons may reach to large distances, causing the Coulomb force to fall off only as $1/r^2$.

2.3 Leptons and their decays

- Leptons are a family of spin 1/2 fermions that interact with other particles through the EM and weak interactions but *not* via the strong interaction. Three leptons and three associated neutrinos (and all their anti-particles) are known: electrons, muons (mass about 200 m_{e}) and taus (mass about 3500 m_{e}).

- Apart from mass, muons and taus appear identical to electrons. In particular, all appear to be completely structureless and thus truly elementary. Electrons, muons, and taus can all annihilate with their anti-particles to create two or three photons; the inverse processes can also occur.

- Apparently the three types of associated neutrinos, ν_{e}, ν_μ, and ν_τ are distinct from one another. Their masses are very close to zero, but (according to the latest results from SNO) *not exactly* zero.

- The muon decays via β-decay to an electron plus a ν_μ and a $\bar{\nu}_{\mathrm{e}}$, conserving energy.

- In all transformations involving leptons, *lepton number* (number of leptons + their neutrinos − anti-particles) is conserved separately for each lepton family, at least to a high degree of approximation. Total electrical charge is also conserved.

- When a muon decays, the electron momentum vector is usually opposite to the spin of the muon. Now, under reflection through the origin, the vectors $\mathbf{r}$ and $\mathbf{p}$ change sign, so $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ and $\mathbf{s}$ do not. So if we reflect the decay through the origin (the parity operation), the electron momentum vector would be parallel to the spin. Thus weak interactions are *not* invariant under the parity operation; parity is not conserved in weak interactions.

3 Nucleons, mesons, quarks and the strong interaction

Hadrons are particles that experience the strong interaction. We shall soon see that the familiar ones are composite bodies, made of more fundamental particles that we call *quarks.*

3.1 Protons, neutrons, and the quark model

- The two familiar hadrons are the proton and neutron, both with masses of about 940 MeV/c^2, or about 1840 time m_e. The neutron is slightly more massive than the proton. Both have spin 1/2. The neutron is uncharged overall, the proton has a total charge of exactly −e.
- Both particles have distributed charge. That of the proton is smeared out over about a mean radius R_p of about 0.8 fm; that of the neutron is neutral overall with a positive core and negative halo, and about the same extent as the proton.
- Both particles have magnetic moments; that of the proton is $2.79(e\hbar/2m_\mathrm{p})$; that of the neutron is $-1.91(e\hbar/2m_\mathrm{p})$. These non-integral coefficients strongly indicate that the proton and neutron are composite, not elementary, particles.
- The composite nature of protons and neutrons is also shown by the complex variation of total photon cross section, very different from the Compton scattering cross section of the electron. The first resonance in nucleon photon cross-sections is at about 294 MeV; at least this much energy is required to excite internal structure in nucleons. In this as in most respects concerned with the strong interaction, the proton and neutron behave almost identically.
- Particle physicists have concluded that protons and neutrons are systems composed of three *quarks* bound together by the *gluon* field. The quarks found in the nucleons are the up (u) and down (d) quark, both spin 1/2 particles with charges respectively +(2/3)e and −(1/3)e. Thus the proton is a (uud) composite while the neutron is a (udd). Such three-quark systems are called *baryons.*
- It is thought that quarks have only small masses compared to nucleons; most of the mass is in the gluon field energy. The d quark is a little heavier than the u quark.

3.2 The nucleon-nucleon interaction; quarks and mesons

- Because the energy required to excite a quark system is about an order of magnitude larger than the potential energies of nucleons bound in a nucleus, it is possible to treat the nucleons as structureless particles – in this context – whose interactions may be studied without looking into their quark nature. (This is like treating the nuclei of atoms as structureless.) The resulting nucleon–nucleon interactions are however expected to be complicated, and they are.

- Information about the interactions between pairs of nucleons may be obtained from studying the deuteron and from scattering experiments of protons and neutrons on protons. targets.

- The nucleon-nucleon potential has been characterized in detail. Two potentials are actually needed, one for the states anti-symmetric under exchange, and one for the symmetric state. For both these situations, two-nucleon combinations with $\mathbf{S} = 0$ experience only a central potential $V(r)$. The $\mathbf{S} = 0$ potentials, although attractive, are not strong enough to lead to any bound state of the two-nucleon system.

- When $\mathbf{S} = 1$, the potential is much more complicated, and differs between the symmetric and anti-symmetric cases. Both cases have terms proportional to $(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r})$, $(\sigma_1 + \sigma_2) \cdot \mathbf{L}$, etc, where $(\hbar/2)\sigma_1$ is the spin operator for nucleon 1. These potentials generally add to produce a net effect like the attractive potential of a molecule: a repulsion at small distances, then attraction at larger distance. The "tensor" term that arises in the symmetrical $\mathbf{S} = 1$ is sufficiently attractive to lead to a bound state of the two-nucleon system.

- The deuteron is the only bound state of two nucleons. A bound n-p state ($E_{\mathrm{B}} = -2.225$ MeV) can exist, while n-n and p-p states do not, in spite of the fact that the strong force is almost the same for p's and n's, because these two *non-identical* particles can form a system with a wave-function which is symmetric under n-p exchange. This state is not accessible to two identical nucleons.

- The gluon field can bind a quark and an anti-quark to form short-lived *mesons* such as the $\pi^+ = (\mathrm{u\bar{d}})$, the $\pi^- = (\mathrm{d\bar{u}})$, and the $\pi^0 = (\mathrm{u\bar{u}} - \mathrm{d\bar{d}})/\sqrt{2}$, with masses near 140 MeV/c^2, and spin and orbital angular momentum also zero. Other heavier mesons (all with integral spin) occur.

- Gluons confine quarks within a nucleon. Mesons are the main fields confining nucleons within a nucleus, particularly the π mesons because their range is of order 1.4 fm.

3.3 Hadrons and the weak interaction

- Hadrons interact via the electromagnetic and weak interactions. Beta-decay, in which a neutron becomes a proton or vice versa, proceeds through the coupling between quarks and the intermediate bosons. Thus a d quark may emit an e^- and a $\bar{\nu}_{\mathrm{e}}$ to become a u quark, converting a proton into a neutron. It is because of the weak interaction that all mesons are unstable.

- There are four further types, or flavours, of quarks known: the strange (s), charm (c), top (t) and bottom (b) quarks. The s and c quarks have charge $-(1/3)$e while the other two have $+(2/3)$e. Many of the more massive particles of high-energy physics are composed of these more massive quarks.
- It appears that baryon number (baryons – anti-baryons) is conserved in all interactions.
- The theory of the strong, EM, and weak interactions sketched above is known as the *Standard Model* of particle physics. It has succeeded in explaining and predicting many phenomena, but it seems to be still rather incomplete. It is in any case very complex, and rests on mathematical methods rather different from those used in atomic quantum physics.
- Fortunately, we will see that many nuclear phenomena may be understood with simpler theoretical models derived from the wave mechanics methods developed to study atoms and molecules.