

ISOTOPE EFFECT IN THE SPECTRUM OF HYDROGEN

PHYSICS 359E

INTRODUCTION

In this experiment, you will measure the ratio of the mass of the electron to that of the proton by observing the wavelength separation of lines in the spectrum of hydrogen and of deuterium. As well, you will determine experimentally the quantum numbers associated with the transitions observed and determine a value for the Rydberg constant.

You may recall from your study of the Bohr theory that the hydrogen spectrum consists of several series of lines whose wavelengths are given by

$$\frac{1}{\lambda} = -R \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \quad (1)$$

where n_i and n_f are the quantum numbers associated with the initial and final energy levels. The best known series are named after spectroscopists from the early part of this century: Lyman ($n_f = 1$), Balmer ($n_f = 2$), Paschen ($n_f = 3$), Brackett ($n_f = 4$) and Pfund ($n_f = 5$). The constant R is known as the Rydberg constant, or simply 'the Rydberg'. In deriving the value for R , it is often assumed that the electron revolves about a stationary nucleus. This is equivalent to assuming that the nucleus has infinite mass and leads to a value R_∞ . Real nuclei obviously have finite mass and the electron and nucleus revolve about their common centre-of-mass. The correct value for the Rydberg in Eq.(1) for an atom whose nucleus has mass M is

$$R_M = \frac{1}{1 + m/M} R_\infty \quad (2)$$

where m is the mass of the electron.

The nucleus of the hydrogen atom consists of a proton of mass M_p , while that of the deuterium has one proton and one neutron so that its mass is very nearly $2M_p$. Combining these facts with Eqs. (1) and (2) leads to the following simple result for the fractional wavelength separation of deuterium lines from the comparable hydrogen lines:

$$\Delta\lambda/\lambda \approx m/2M_p. \quad (3)$$

PRE-LABORATORY PREPARATION

1. Review Bohr theory by reading Sec. 1.4 in Ref. 1.
2. Derive Eq. (3) from Eq. (2); include this derivation in your lab report.
3. Read Secs. 1.5 and 4.6 in Ref. 1 to acquaint yourself with grating spectrographs and the Fabry-Perot interferometer.
4. After reading this write-up, make a table of grating settings (values of the angle θ) for which the blue-green line of H ($H\beta$) should be visible, and of the linear dispersion of the spectrograph for each setting for which $H\beta$ is visible. Include this table in your report.

GENERAL PROCEDURE AND EQUIPMENT NOTES

The light sources for this experiment are hydrogen and hydrogen-deuterium lamps. These sources emit hydrogen lines strongly as well as diffuse bands due to molecular hydrogen. You will notice that the light from the lamp is purplish; this is because the two strongest lines which it emits in the visible spectrum are red and blue. All the measurements you make will be on these two lines. Filters are available to isolate the lines.

Wavelength measurement with the Fabry-Perot interferometer

The Fabry-Perot interferometer consists of two parallel partially-reflecting mirrors and a device for varying their separation t . To understand the basic idea of its function, we use the special case of normally incident light. Consider the interference between light which passes through the instrument directly and that which is reflected back and forth once, as seen in Fig. 1. Constructive interference will occur whenever the path difference between the two paths is an integral multiple of the wavelength of the light:

$$m\lambda = 2t, \quad m = 0, 1, 2, \dots$$

The general theory for light that is not normally incident is discussed in Ref. 1, Sec. 7.5 (which you are to read), and in Ref. 5. If you increase t slowly, each time it changes by $\lambda/2$, the on-axis intensity will change from bright to dark to bright again. Thus all you have to do is measure the change in t for n such fringes. Since there will always be an uncertainty of about one fringe in your count, the precision of the wavelengths obtained will depend on how many fringes you count. To get the wavelength to 1 part in 1000, you would have to count 1000 fringes.

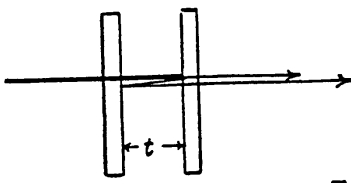


Figure 1: The light path through a Fabry-Perot etalon.

Let's assume you've finished the measurements and have found the wavelengths of the lines to be λ_{red} and λ_{blue} . From Eq.(1) one can write

$$\frac{1}{\lambda_{\text{red}}} = -R_{\text{H}} \left(\frac{1}{n_{\text{i}}^2} - \frac{1}{n_{\text{f}}^2} \right) \quad \text{and} \quad \frac{1}{\lambda_{\text{blue}}} = -R_{\text{H}} \left(\frac{1}{m_{\text{i}}^2} - \frac{1}{m_{\text{f}}^2} \right)$$

where the n 's and m 's are the quantum numbers appropriate to the transitions. At first glance, it appears to be impossible to solve these since there are only two equations and five unknowns. However, four of the unknowns are integers and this fact may allow you to solve for them provide that your measurements are sufficiently precise (and the integers sufficiently small).

The ratio of the wavelengths is given by

$$\lambda_{\text{red}}/\lambda_{\text{blue}} = \left(\frac{1}{m_{\text{i}}^2} - \frac{1}{m_{\text{f}}^2} \right) / \left(\frac{1}{n_{\text{i}}^2} - \frac{1}{n_{\text{f}}^2} \right),$$

and the problem becomes finding what sets of the four integers give a value for the ratio which agrees with the experimental value within the experimental error. If more than one set satisfies the

equation, it may be possible to eliminate all but one by careful reasoning. If no set satisfies the equation, either Bohr theory or your measurements must be at fault! Once the transitions involved are determined it is straightforward to determine R_H .

Wavelength separation measurement using a grating spectrograph

To measure the separation of the closely spaced lines emitted by ^1H and ^2H , a Fabry-Perot interferometer is ideal in principle; however, the one available in the lab isn't quite suitable, so a grating spectrograph will be used instead. The instrument, a Bass-Kessler mounting (see J.O.S.A. article available in lab), consists of a slit, two camera lenses, a plane reflection diffraction grating and either a 35mm camera back, or a CCD (charge-coupled device) electronic detector. The slit is in the focal plane of the first (*collimator*) lens so that the grating is illuminated in parallel light. The second (*camera*) lens form images of the slit on detector (film or CCD), as shown in Fig. 2.

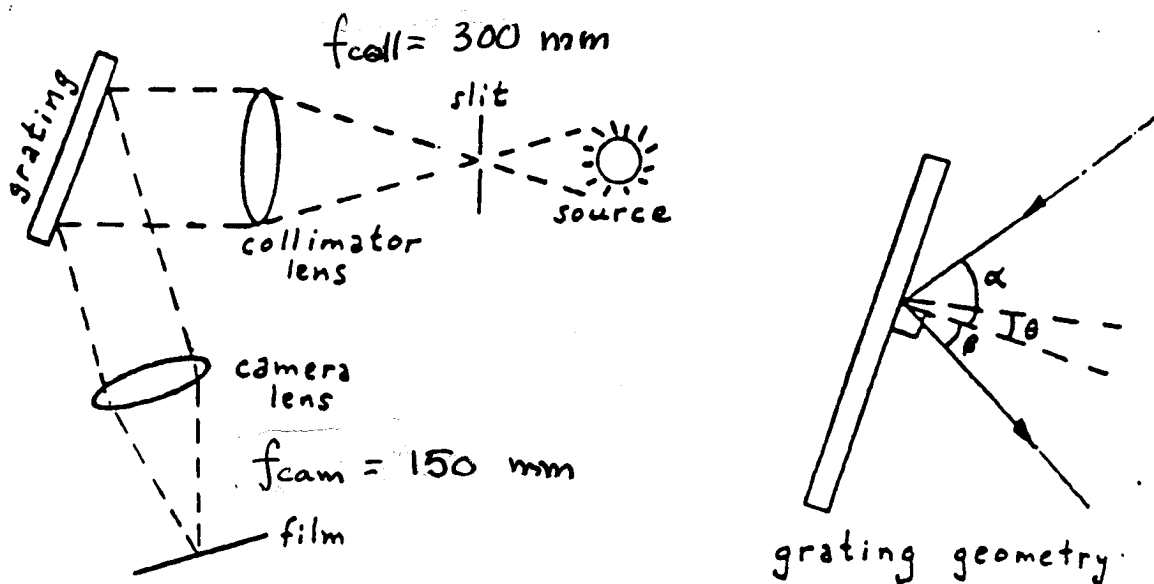


Figure 2: The light path and basic angles describing a grating spectrograph.

The basic grating equation is

$$n\lambda = a(\sin \alpha - \sin \beta) \quad (4)$$

where n = order, a = linespacing = $0.83333 \mu\text{m}$, α = angle of incidence, and β = angle of diffraction. This form is not particularly convenient for use with this mounting. Since the alignment of the optic axes of the lenses is fixed, $\alpha + \beta$ is fixed. Defining $\alpha + \beta = 2\varphi$ and $\alpha - \beta = 2\theta$, we can reformulate the equation as

$$n\lambda = 2a \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2} = 2a \sin \theta \cos \varphi \quad (5)$$

Here θ is the angle between the grating normal and the bisector of the two optic axes, and 2φ is the angle between the two optic axes. In this instrument, $\varphi = 27.5^\circ$. From the diagram it should be clear that the maximum value of θ available with this mounting is $\pm(90^\circ - 27.5^\circ) = \pm 62.5^\circ$, which clearly sets an upper limit on $n\lambda$.

In order to find the wavelength separation of the two isotopic lines on the detector, we need to know the *linear dispersion* $d\lambda/dx$ of the instrument, where x measures linear displacement along the spectrum. We can find the *angular dispersion* $d\lambda/d\beta$ simply from the basic grating equation:

$$d\lambda/d\beta = (a \cos \beta)/n,$$

and geometric optics gives $dx = f_{\text{cam}}d\beta$, where f_{cam} is the focal length of the camera lens (equal to 150 mm, *not* 300 mm as stated in the article on the spectrograph; the collimator focal length is 300 mm), so that

$$d\lambda/dx = (a \cos \beta)/(f_{\text{cam}}n).$$

Note that since the dispersion depends on $\cos \beta$, it will not be the same in the $+n^{\text{th}}$ order as it is in the $-n^{\text{th}}$ order. To find $d\lambda/dx$, it will be necessary to find θ from Eq.(5) and then β from $\beta = \varphi - \theta$.

PROCEDURE

1. The two mirrors of the Fabry-Perot interferometer must be aligned internally to make them accurately parallel. Remove the telescope from the interferometer and orient the He-Ne laser so that the beam is normally incident on the fixed mirror; you can use the spot reflected back toward the laser to do this. Now observe the transmitted laser beam on the wall and align the movable mirror until all spots are superposed as closely as possible. As the two plates approach good alignment, you should see flickering of the spot due to interference as the spacing is changed.
2. In order to see a ring pattern, the interferometer must be illuminated with a diffuse light source. Why? Diverge the laser beam with a lens and place a tissue between the lens and the interferometer. Look through the telescope for interference fringes. You will probably need to tweak the alignment delicately to get the best fringes. Record the change in separation of the mirrors as indicated by the differential micrometer for about 250 fringes. From the known air wavelength of the He-Ne red line (632.82 nm), calibrate the micrometer motion.
3. For the next part of the experiment, you will need the interferometer aligned even better than it already is. Install a *helium* lamp into its holder, and focus an image of the lamp roughly onto the first plate. You should see rather bright rings even though you are looking at several lines at once. Now tune the interferometer again, just a little, for ring sharpness and for high contrast between the bright and dark regions. (If you move the screws too far, you may have to start right back with the laser again.)
4. Now exchange the helium tube for a (normal) hydrogen tube. If the interferometer is well enough aligned, you should see rings from this lamp faintly but clearly. Isolate the hydrogen red line with a filter and measure the change in separation of the mirrors as indicated by the differential micrometer for about 250 fringes. Repeat for the blue line if you can see it. It is useful to record the separation every 50 fringes or so; then a plot of separation vs. fringe number will reveal a miscount. Repeat as necessary until you are satisfied that you have determined the wavelengths to one part in 250.
5. Determine the quantum numbers of the lines. Your report should include an energy level diagram of hydrogen drawn to scale and properly labeled with the observed transitions.

6. To align the spectrograph, start with the slit open wide and the focus of both lenses at infinity. (Warning: the spectrograph has a small lever near the slit that *closes the slit*. Find this and make sure that it is open.) Position the H-D light source to maximize illumination on the grating; you may want to use a transfer lens to make the lines visible while you are aligning everything. Install the 35-mm camera on the spectrograph camera. With the camera shutter timer set to B, depressing the shutter release should open the shutter, and the set screw can be used to hold it open indefinitely. You should now be able to view the diffracted image of the slit by eye as a real image in the focal plane of the camera and make adjustments to achieve a narrow, well-focused image. (The view-finder has a small flip-down magnifying glass in it which may be useful.) You should be able to clearly see the blue-green $H\beta$ line of hydrogen in several orders, at approximately the grating angle setting that you calculated before coming to class. Check your grating angle calculations experimentally and include comments on this in your report.
7. The linear dispersion is quite different in the different orders that you can observe. In one of these orders, you should be able to see that the $H\beta$ line is actually split into two components, due to H and D. You will need to have the entrance slit set fairly small for the two lines to be clearly separated. (Why?) Is the order where you can see this effect the one for which you calculated the largest linear dispersion?
8. Carefully centre a spectral line in the field of view of the camera, not using the magnifier. Now replace the camera with the small computer-controlled CCD detector. You will need to reduce the intensity of the light falling onto the CCD to a level far below what is needed for you to see the line by eye, as the CCD is very sensitive: try removing the transfer lens between the lamp and the entrance slit. Tune your set-up until you are able to record a CCD image of the spectral line which has about 100 ADU amplitude at peak, taking more than 10 s, in which you can see the splitting of the spectral line you are observing. The spectral lines should be aligned parallel to the rows or columns of the CCD for ease of measuring separations. Record an image. Repeat this operation for each line in any order that you can detect splitting in.
9. Measure the line separations in pixels of the lines on each of your images, convert these values to separations in mm using the fact that the CCD pixels are $13.5 \mu\text{m}$ across, and then use your calculated dispersions to convert your result to a separation in wavelength. Finally, calculate a value for m/M_p , and calculate R_H from the theoretical value of R_∞ . In your report, estimate the uncertainty in m/M_p , and compare your value to the accepted one (if you are careful, you should be able to get within a few percent of the accepted value).

REPORT

Your report should include the following:

- Your derivation of Eq. (3).
- Your table of predicted grating settings and dispersions for the various orders in which $H\beta$ is visible.
- The wavelengths of $H\alpha$ and $H\beta$, and your determination of the quantum numbers of the levels which lead to these lines.

- Comments on how well your theoretical grating settings worked for finding $H\beta$, and for predicting the correct order to use for measuring the H–D line splitting.
- A discussion of your measurement of the H–D line splitting, including an estimate of uncertainty.
- Your derived values of m/M_p and R_H , *with uncertainties*, and a comparison to accepted values.

REFERENCES

1. Melissinos, A. C. and Napolitano, J., “Experiments in Modern Physics”, Secs. 1.4, 1.5, 4.6, and 6.3.3.
2. D. Halliday, R. Resnick and K. S. Krane, “Physics”, extended version (John Wiley & Sons, 1992), Secs. 51-1 and 51-2.
3. R. Eisberg and R. Resnick, “Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles” (Wiley, New York, 1985), Ch. 4.
4. E. Hecht, “Optics” (Addison-Wesley, 1987).