Stellar Structure and Evolution

Theoretical Stellar Models

Consider each spherically symmetric shell of radius *r* and thickness *dr*. Basic equations of stellar structure are:

(1) Hydrostatic equilibrium

$$4\pi r^2 dP = -\frac{GM(r)\rho(r)4\pi r^2 dr}{r^2}$$

$$\Rightarrow \quad \frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}.$$

(2) Mass continuity $dM = \rho(r)4\pi r^2 dr \Rightarrow \frac{dM}{dr} = 4\pi r^2 \rho(r).$

(3) Energy transport

Radiative transport

$$F = \sigma T^4 \Rightarrow dF = 4\sigma T^3 dT.$$

Also $dF = -F(\tau)d\tau = -\kappa(r)\rho(r)F(r)dr.$

Combine, use $L(r) = 4\pi r^2 F(r)$ and add proper constants

$$\Rightarrow \quad \frac{dT}{dr} = -\frac{3\kappa(r)\rho(r)}{64\pi\sigma r^2 T^3(r)} L(r),$$

But, if κ is high enough, transport becomes convective,

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T(r)}{P(r)}\frac{dP}{dr}, \text{ where } \gamma = 5/3.$$

(4) Energy generation

$$dL = 4\pi r^2 dr \rho(r)\varepsilon(r),$$

where ε, in units J kg^{-1} s^{-1} is the energy source.

$$\frac{dL}{dr} = 4\pi r^2 \rho(r)\varepsilon(r).$$

(5) Equation of state

Perfect gas law $P = nkT$.

$$n(r) = \frac{\rho(r)}{\mu(r)m_H}, \text{ where } \mu(r) = \frac{1}{2X + 3/4Y + 1/2Z} \text{ is the "composition"}$$

$$\Rightarrow P(r) = \frac{k\rho(r)T(r)}{\mu(r)m_H}.$$

Energy Sources (e)

For pre-main-sequence stars, ε from gravitational contraction.

For main-sequence and giant stars, ε from nuclear reactions.

For solar type stars, ε mainly from PP I chain (see p. 315 in text),

$$e = \text{constant} \times X^{2} r T^{4}.$$

For higher temperature stars, ε from CNO cycle (see p. 316 in text).

Opacity

κ(ρ,T) mainly due to free-free or bound-free interactions.. In this case, use Kramer’s opacity

$$k = \text{const.} \times Z(1+X)\frac{r}{T^{3.5}}.$$

Calculations of more general opacities including all possible interactions is very complex.

Summary of Principles of Stellar Structure

(1) Hydrostatic equilibrium: A star is in mechanical equilibrium with the pressure at every level equal to the weight of a column of material per unit cross-sectional area on top.

(2) Energy transfer: Photons in the interior carry energy outward by random walking from regions of higher temperature to regions or lower temperature. If the luminosity required to be carried out is too large for this process, convection results.

(3) Energy generation: Energy release in the interior, through nuclear reactions, balances the outward release of energy through radiation or convection. If the nuclear source is inadequate, gravitational contraction must occur.

Stellar Evolution Calculations

Start with initial hydrostatic equilibrium model with uniform composition (μ = constant). Allow energy generation ε to occur.

$$e(r) \quad \Rightarrow \quad \text{changes composition } m(r)$$

$$\Rightarrow \quad \text{changes opacity } k(r) \quad \Rightarrow \quad \text{affects } P(r), \frac{dT}{dr}, \text{etc.}$$

$$\Rightarrow \quad \text{star adjusts to new equilibrium}$$

$$\Rightarrow \quad \text{affects } e(r), \quad \text{so back to first step....}$$

Therefore, a star evolves in time due to nuclear reactions.

Key point: It is found that the evolution of stars differ only due to their mass *M* and initial composition μ.

Physical Basis of Mass-Luminosity Relation

From stellar structure equations,

$$\frac{dP}{dr} = -\frac{GM(r)\,r(r)}{r^2} \Rightarrow \frac{0 - P_c}{R} \propto \frac{Mr}{R^2} \Rightarrow P_c \propto \frac{Mr}{R}.$$

For a perfect gas, $P_c \propto rT_c \Rightarrow rT_c \propto \frac{Mr}{R} \Rightarrow T_c \propto \frac{M}{R}.$

Same approximation to radiative energy transport equation

$\Rightarrow L \propto \frac{R^4 T_c^4}{k\,M}.$ Therefore, $L \propto \frac{R^4}{k\,M}\left(\frac{M}{R}\right)^4 \propto \frac{M^3}{k}.$

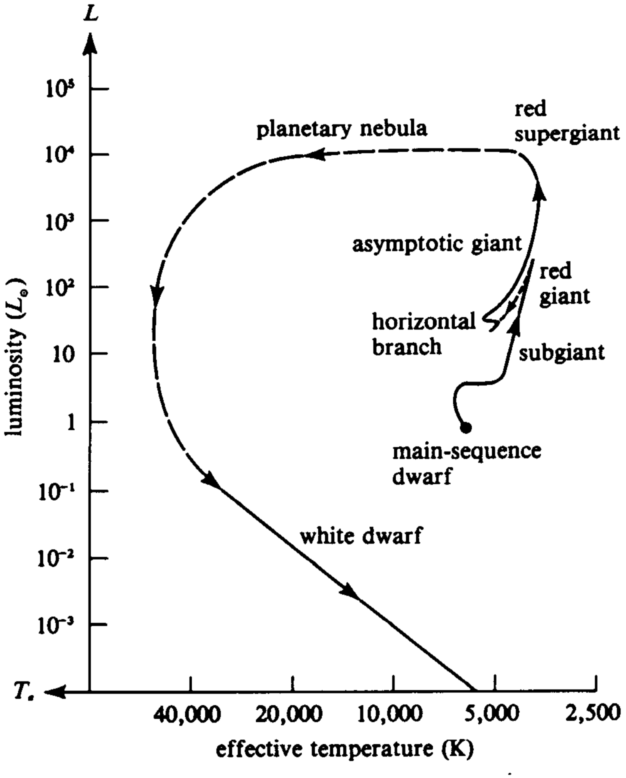
Compare to observed average for main-sequence, $L \propto M^{3.3}.$

Conclusion: Massive stars are much more luminous but have much shorter lifetimes,

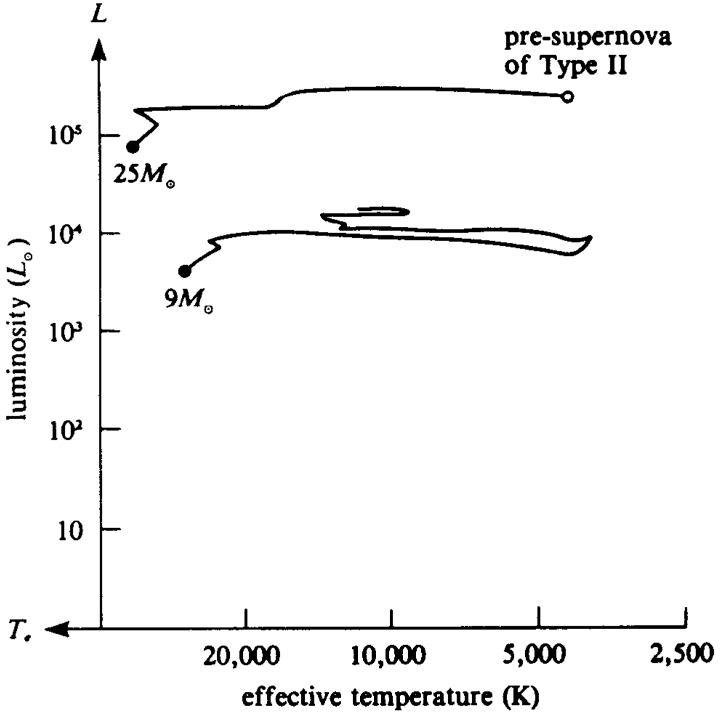
$t \propto \frac{M}{L} \propto M^{-2}k$ (or $M^{-2.3}$ empirically).

Stellar Evolution

Theoretical evolutionary tracks on H-R diagram.



1 M_{Sun} object. Dashed lines denote uncertain phases.



9 and 25 M_{Sun} objects.

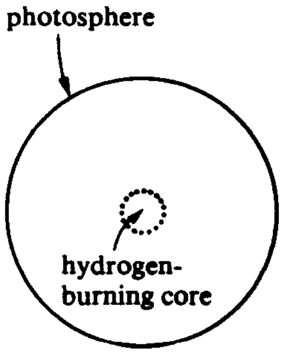
Evolution of a 1 M_{sun} (Pop I) Star

Zero-age main sequence

Core H burning begins

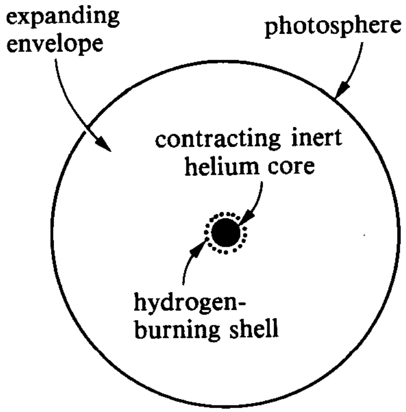
Evolution on main sequence

Core H fused to form He. Accounts for most of the star’s nuclear burning lifetime



Subgiant phase

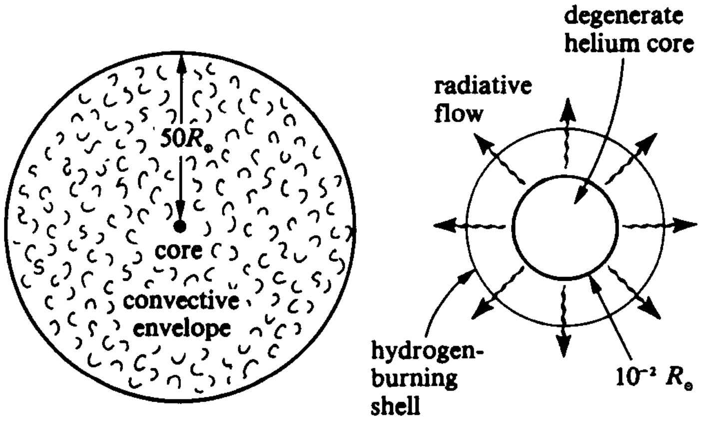
Core H exhausted => He core; H burning in shell begins; envelope expands, core contracts



(b) post-main sequence

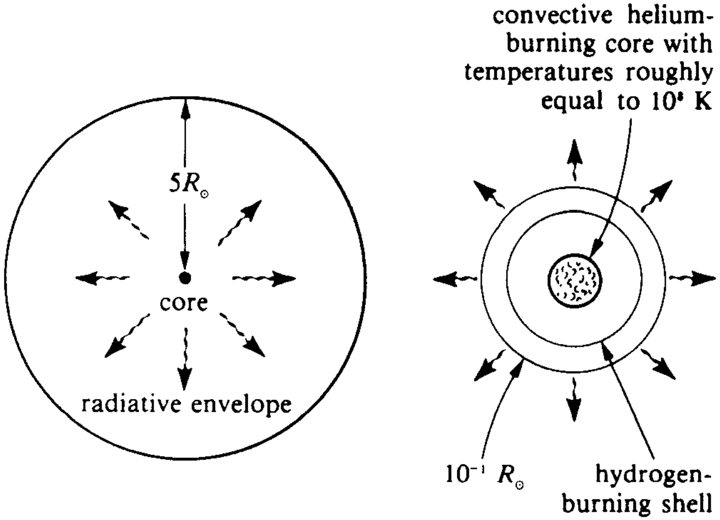
Evolution of a 1 M_{sun} (Pop I) Star

Red giant phase



Envelope continues to expand; luminosity increases; envelope becomes convective due to increasing opacity; core supported by electron degeneracy pressure, a Quantum mechanical effect

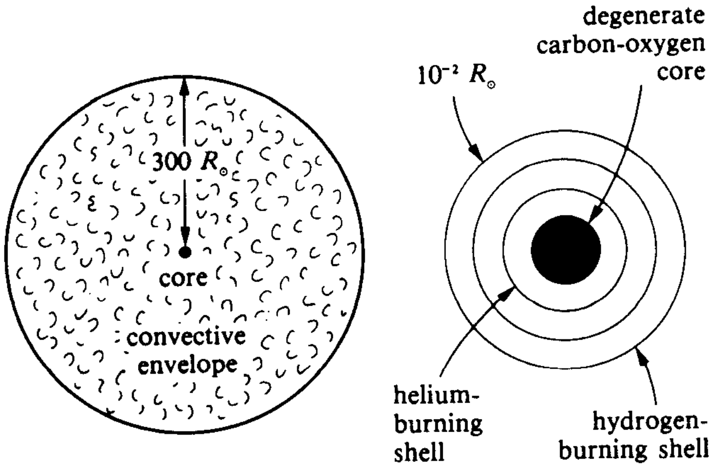
Horizontal branch



He burning begins in core; starts with a "flash" due to degeneracy support; H shell burning continues; star settles into a "stable" analog of main sequence

Evolution of a 1 M_{sun} (Pop I) Star

Asymptotic giant branch



He in core exhausted; He and H burning in successive shells (double-shell burning); core shrinks, envelope expands again

Thermal pulses (variable star)

Sensitivity of He shell burning to temperature leads to cycles of expansion and contraction

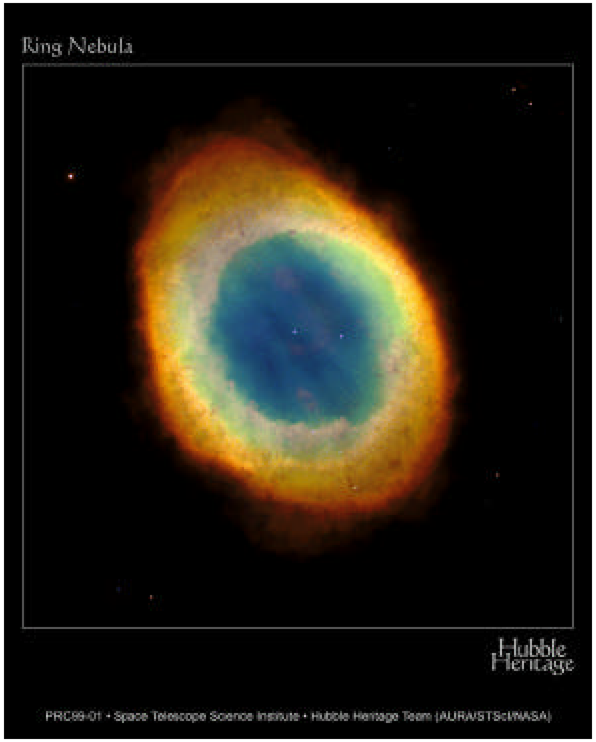
Evolution of a 1 M_{sun} (Pop I) Star

Planetary nebula

Pulsations lead to superwind which ejects stellar envelope; see central star and ejected halo

White dwarf

Core does not reach ignition temperature of C; all thermonuclear reactions stop; slow cooling



The Ring Nebula - a planetary nebula.

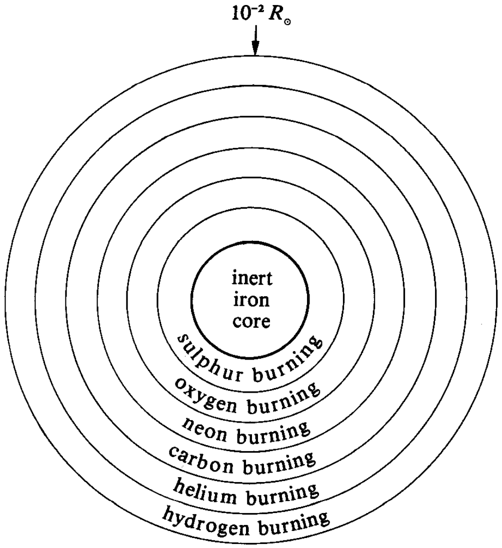
Evolution of Very Massive Stars

Occurs extremely rapidly, due to high energy generation rate.

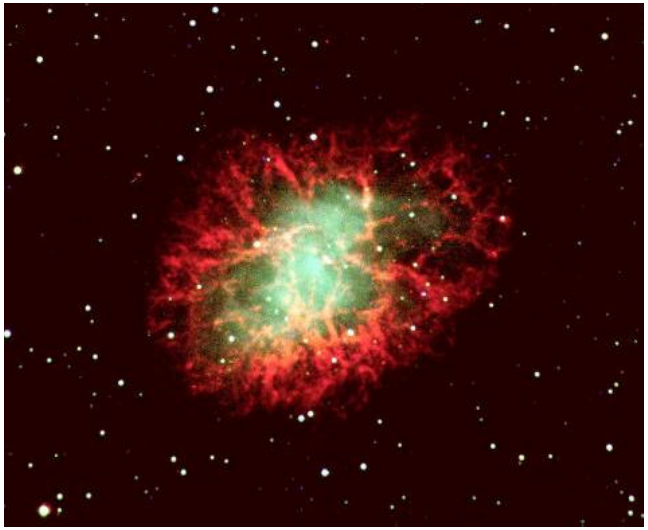
Go through successive periods of core exhaustion of a fuel followed by ignition of the next heavier element. Fusion continues beyond carbon.

Outer layers do not have much time to respond to changes in core; only a steady drift to the right in the H-R diagram.

Eventually, reach iron catastrophe . Nuclear fusion cannot release energy in iron core. Core collapses => rebound at nuclear densities can lead to a supernova (SN).



Core of a pre-supernova star



SN remnant - The Crab Nebula

Some Conclusions From Stellar Evolution Calculations

- Evolutionary history depends on only two parameters: mass M and initial composition μ.

- $M = 0.08\ M_{sun}$ is the minimum mass to initiate nuclear reactions.

- Very low mass stars ($0.08 < M/M_{sun} < 1$) have main-sequence lifetimes longer than the estimated age of the universe.

- The Sun's main-sequence lifetime is about 10^{10} yr.

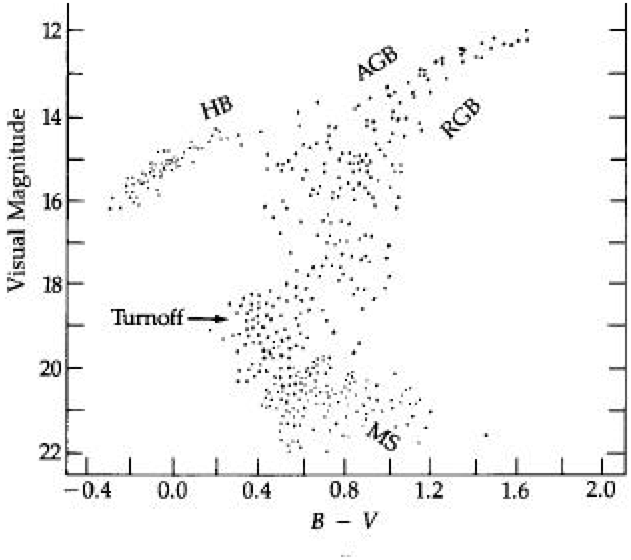
- Very high mass stars can have a main-sequence lifetime of only a few x 10^6 yr.

- A star gets *hotter* as it evolves, even though energy is continually radiated away. This is a gravitational effect.

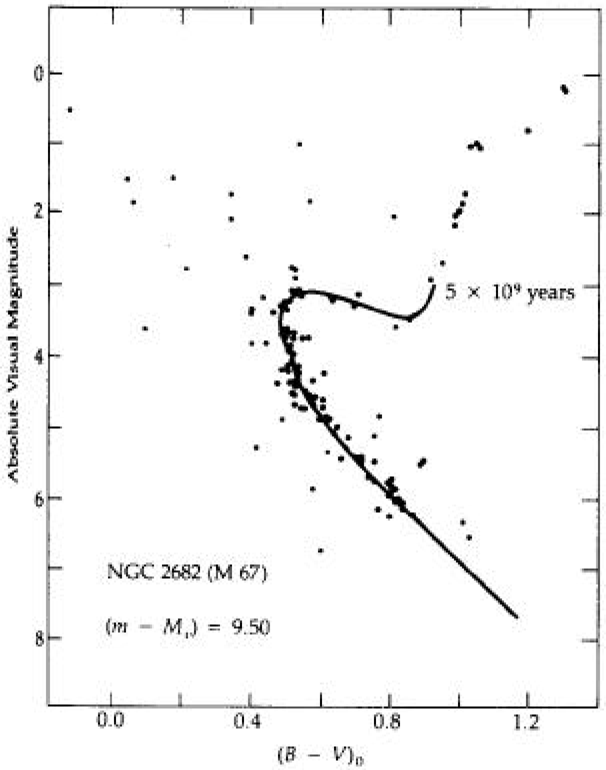
- The Sun is stable during nuclear burning. An increase in ε => expansion => lower T_c => decrease in ε.

The Main-Sequence Turnoff

A cluster of stars should exhibit a turnoff point in the main-sequence. More massive stars of a given age have evolved off the MS. Comparison of theory and observations yields a cluster age.



Schematic H-R diagram of a globular cluster with phases labeled.



Observed H-R diagram and theoretical isochrone for 5x10^9 yr.