Star Deaths

Main sequence stars evolve due to continual competition between the tendency toward

(1) Thermal equilibrium <=> energy loss to surroundings

(2) Mechanical equilibrium <=> gravitation <=> center gets hotter to maintain equilibrium.

Eventually, nuclear reactions cease and the required central temperature (and pressure) cannot be maintained =>

Star collapses! To what?

Final Outcome

(1) White dwarf - electron degeneracy pressure (QM effect) supports star

(2) Neutron star - neutron degeneracy pressure (also a QM effect)

(3) Black hole - gravity wins out!

Origin of Degeneracy Pressure

(1) Pauli exclusion principle

No two fermions (e, p, n,…) can occupy the same quantum state

(2) Uncertainty principle $\Delta x \, \Delta p_x > h$

$$p_x \sim \Delta p_x > \frac{h}{\Delta x} \text{ is large,}$$

i.e., high density => high random motions => pressure P.

Note that $P = n\langle v_x p_x \rangle = \frac{1}{3} n \langle vp \rangle$ in general.

"Degeneracy pressure" can exceed kinetic pressure $P=nkT$ in very dense objects; also, does not depend on temperature.

White Dwarfs

Combine the generic relation $P_c \sim \frac{GM^2}{R^4}$

with the nonrelativistic degenerate electron pressure equation

$$P \sim n v_x p_x \sim r^{5/3} \propto (M / R^3)^{5/3}$$ to obtain $R \propto M^{-1/3}$.

A more exact calculation yields the mass-radius relation

$$R = 0.114 \frac{h^2}{G m_e m_p^{5/3}} \left(\frac{Z}{A} \right)^{5/3} M^{-1/3}.$$

More massive white dwarf => smaller radius!

Hints at the existence of a maximum mass.

Limiting Mass of a White Dwarf

At extremely high densities, electrons become relativistic ($v \sim c$).

$$P \sim n v_x p_x \sim n c p_x \propto r^{4/3}.$$

Now find a maximum possible mass $$M \sim \left(\frac{hc}{G}\right)^{3/2}\left(\frac{Z}{Am_p}\right)^2.$$

A more exact calculation yields the Chandrasekhar limiting mass

$$M_{Ch} = 0.20\left(\frac{hc}{G}\right)^{3/2}\left(\frac{Z}{Am_p}\right)^2.$$

For $\frac{Z}{A} = 0.5$, get $M_{Ch} = 1.40\, M_{Sun}$.

Observations of White Dwarfs

Average properties:

$$\langle M \rangle \approx 1 M_{Sun}, \langle R \rangle \approx 10^{-2} R_{Sun}$$

$$\Rightarrow \langle r \rangle \approx 10^{6}\, r_{Sun} \approx 10^{9} \text{ kg/m}^{3}.$$

How observed?

Example: Sirius B, the binary companion to Sirius

Kepler's 3rd Law => $M = 1.05\ M_{\mathrm{Sun}}$.

Also T = 29,500 K but $L = 3 \times 10^{-3}\ L_{\mathrm{Sun}}$ => $R = 7 \times 10^{-3}\ R_{\mathrm{Sun}}$

=> $\rho = 3 \times 10^{9}$ kg/m^3, can only be a white dwarf!

Luminosity of a White Dwarf

Visible white dwarfs have residual thermal energy - “cooling embers”

Luminosity from ion thermal energy converted to radiation.

WD radiates => loses thermal energy => cools but does not contract

=> given enough time, comes into thermal equilibrium with surroundings. Becomes a <u>black dwarf</u>.

Neutron stars

Consider a collapsing star with M > M_{Ch}: electron pressure P_e cannot stop collapse. As density rises, electrons forced inside nuclei: e + p $\longrightarrow$ n. Yields a self-gravitating mass of neutrons.

Neutrons are fermions => degeneracy pressure.

As for WD's, get a mass-radius relation

$$R = 0.114 \frac{h^2}{G m_n^{8/3}} M^{-1/3}.$$ Plug in #'s =>

$$M = 1.4 M_{Sun} \Rightarrow R = 1.35 \times 10^4 \,\mathrm{m} = 13.5\,\mathrm{km} \Rightarrow \langle r \rangle = 2.7 \times 10^{11}\ \mathrm{kg/m}^3.$$

Compare with atomic nucleus density, $\dfrac{m_p}{4/3p\left(10^{-15}\,\mathrm{m}\right)^3} = 4 \times 10^{11}\ \mathrm{kg\ m}^{-3}.$

Therefore, neutron star <=> a giant nucleus held together by gravity.

Other Properties of Neutron Stars

(1) Very strong gravity. Previous numbers yield

$$v_{esc} = \left(\frac{2GM}{R} \right)^{1/2} = 1.66 \times 10^{8} \text{ m/s} = 0.55c\,! \quad \text{General relativity required.}$$

(2) Upper mass limit

Exists for same reason as for WD's (saturation of degeneracy pressure when $v \sim c$). However, upper limit uncertain due to incomplete understanding of nuclear force.

Current best estimate, $M_{NS} \leq 3M_{Sun}$.

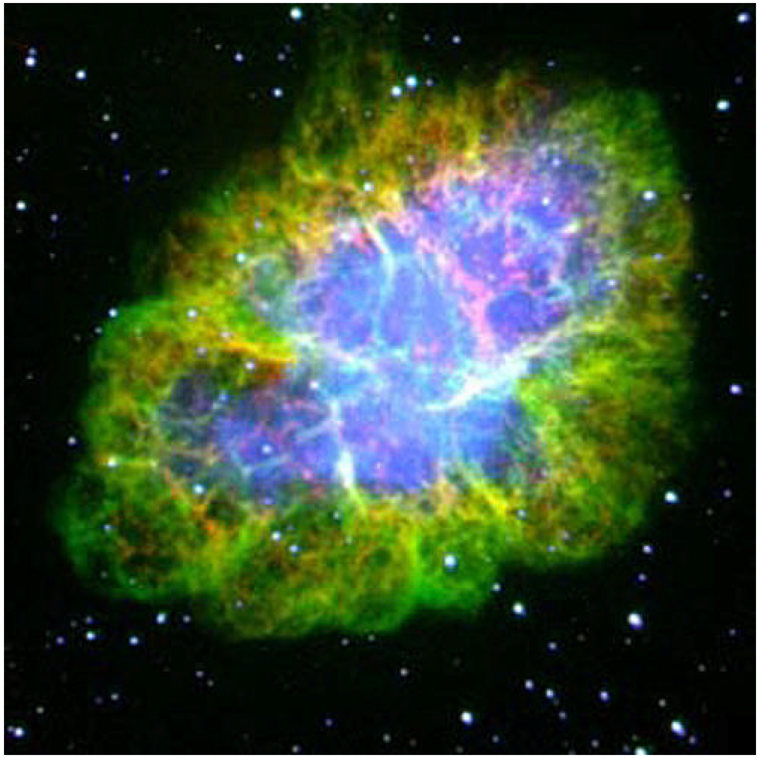
Pulsars

Discovered as pulsating radio sources, 1967-1968. Some 500 detected now.

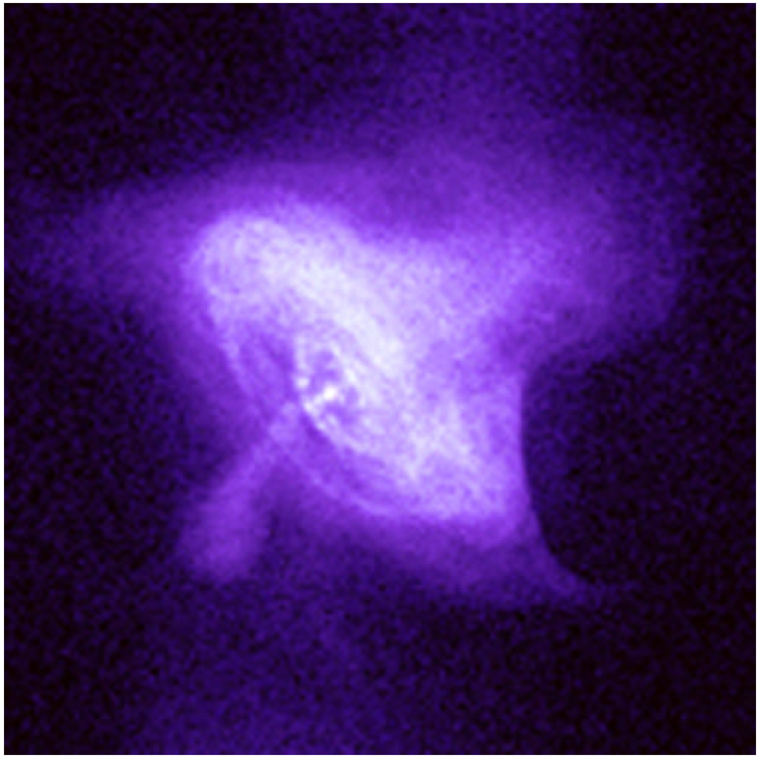
<u>Some properties</u>:

- Periods $P = 1.5$ ms - 3 s. Time between pulses constant to within one part in 10^8.
- Few optical identifications, most notably the Crab nebula pulsar and Vela nebula pulsar; both are supernova remnants. Crab pulsar exhibits same P in gamma rays, X-rays, optical, and radio.
- Pulsars identified with rotating, magnetized neutron stars.

The Crab Nebula and Pulsar



Optical image from Mt. Palomar 5m telescope



X-ray image of center from Chandra telescope

Is a Pulsar a Neutron Star?

Yes. If the pulse period *P* corresponds to a rotation period, then breakup can be avoided only if

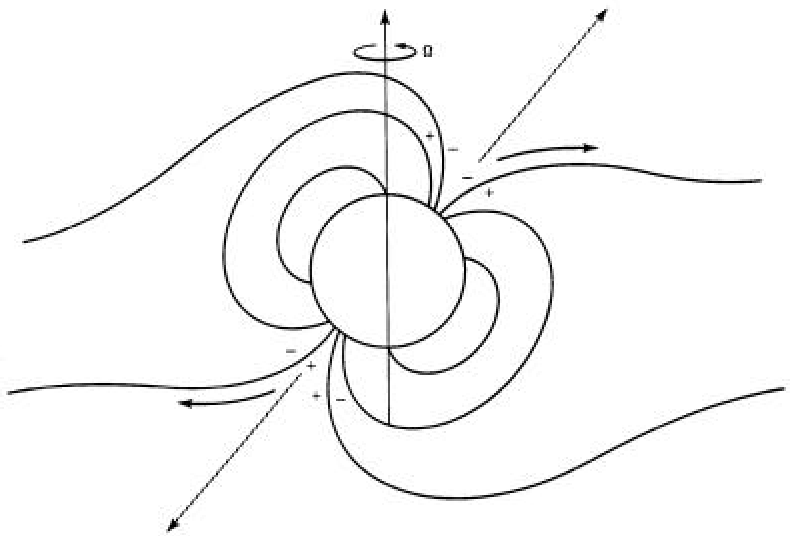
$$\frac{v^2}{R} \le \frac{GM}{R^2}. \qquad \text{Combine with}$$

$$P = \frac{2p\,R}{v} \;\text{ and }\; M = \frac{4}{3}p\,R^3 r \qquad \text{to yield}$$

$$r \ge \frac{3p}{G}\frac{1}{P^2}.$$

Therefore, *P* = 1 s => ρ > 1.4 x 10^{11} kg m^{-3}, a typical NS density!

Emission Mechanism for Pulsars



"Lighthouse model"

Magnetic axis slightly misaligned with rotation axis.

Spinning ***B*** => enormous ***E*** field, which pulls charged particles off surface. Accelerated particles emit radiation => see pulse if/when beam comes into view. Torque from departing particles also slows down pulsar period P.

Measure $B \sim 10^8$ T, enough to explain required E, and $dP/dt \sim 10^{-5}$ s/yr for Crab pulsar, resulting in enough (rotational) energy loss from pulsar to explain emitted energy from Crab nebula.

Black Holes

Stellar remnants of mass > 3 M_{Sun} go into indefinite collapse => to a singular point of zero volume and infinite density, a singularity.

However, there exists an important length scale, where

$$v_{esc} = \left(\frac{2GM}{R} \right)^{1/2} = c \quad \Rightarrow R_{Sch} = \frac{2GM}{c^2}, \quad \text{Schwarzschild radius}$$

$$R_{Sch} = \frac{2GM_{Sun}}{c^2} \frac{M}{M_{Sun}} \cong 3 \frac{M}{M_{Sun}} \text{ km.}$$

Note: smaller than NS

Can calculate R_{Sch} for any object.

$R > R_{Sch}$: radiation and matter can escape

$R = R_{Sch}$: radiation just escapes

$R < R_{Sch}$: nothing escapes

Physical Meaning of Schwarzschild Radius

Outside observer cannot detect any events within this surface, known as the event horizon.

Journey into a black hole:

(1) Strong tidal forces rip objects apart, since Roche limit d is typically much greater than R_{Sch}.

(2) Observer on mass element crosses event horizon without incident on way to crashing into the singularity. External observer never sees the matter traverse the event horizon. Matter is apparently held up at the boundary forever; a time-dilation effect.

Observational Signatures

See radiation from matter as it falls in toward event horizon.

$$L \approx \frac{d(PE_{grav})}{dt} \approx \left(\frac{GM}{R_{Sch}}\right)\frac{dm}{dt}.$$

Can explain extremely high luminosities from

(1) Some binary systems, e.g., Cygnus X-1. See X-ray emission presumably from material falling onto low mass black hole $M \geq 3M_{Sun}$. $L \approx 10^{30}$ W $\Leftrightarrow dm/dt \approx 10^{-9} M_{Sun}/\mathrm{yr}$.

(2) Centers of very high luminosity active galaxies and quasars, presumably from material falling onto supermassive black hole

$$M \geq 10^{6} M_{Sun}.$$

Observational Signatures

The relation between Galactic X-ray sources and BH/NS:

Example: observe point-like source of luminosity $L \sim 10^{30}$ W, with peak emission in X-ray band, at 0.3 nm.

Wien's Law => $T \approx 10^{7}$ K.

$$\therefore\ R = \left(\frac{L}{4 p\, s\ T^4} \right)^{1/2} \approx\ 10\,\mathrm{km},$$

comparable to the size of a neutron star or R_{Sch} of a black hole.