Equilibrium Properties of Matter and Radiation

Temperature

What is it?

A measure of internal energy in a system. Measure from

(1) velocities of atoms/molecules

(2) population of excited/ionized states

(3) properties of emitted radiation

Maxwell Distribution for Gas Velocities

From a microscopic point of view, temperature (*T*) is a measure of the velocity distribution of gas particles.

At equilibrium, gas particles obey a distribution function for speeds

$$F(v) \propto v^2 e^{-1/2 mv^2/kT}$$

Maxwellian Distribution

where *m* = mass of individual particles, *k* = Boltzmann constant, and *F(v) dv* is the probability that a gas particle has speed in the interval *dv*.

Mean speed? Most meaningful is the root mean square value

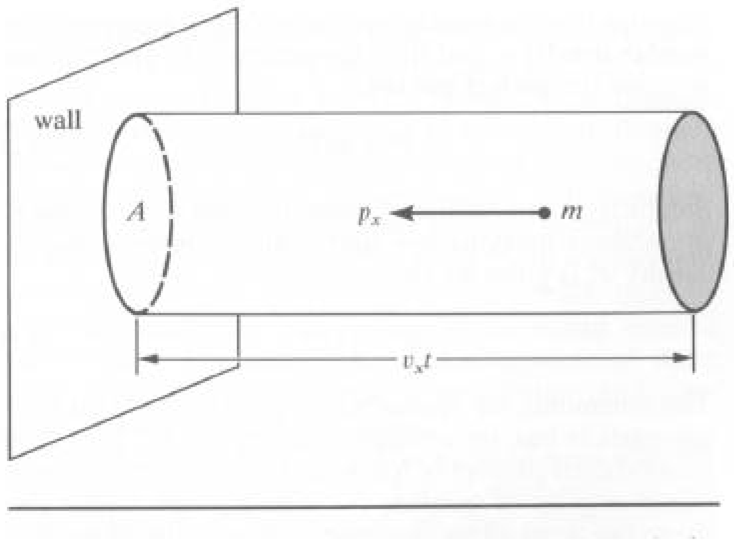
$$v_{rms} = \langle v^2 \rangle^{1/2} = \left(\frac{3kT}{m} \right)^{1/2}.$$

T from this definition known as the <u>kinetic temperature</u>.

Ideal (Perfect) Gas

$P = n\,k\,T$, where P = pressure = force/area, and n = number density.

Derive above relation from definition of pressure, properties of Maxwell distribution, and diagram below.



$$P = n\langle v_x p_x \rangle = \frac{1}{3} n \langle m v^2 \rangle = n\,k\,T.$$

Excitation Equilibrium

The equivalent width of a spectral line depends on the number of atoms in the energy state from which the transition occurs.

Level populations depend on *T*: high *T* => more KE to cause excitations. In steady state (excitations balanced by de-excitations),

$$N_B / N_A = (g_B / g_A)\exp\left[-(E_B - E_A)/kT\right]$$ Boltzmann equation

g = multiplicity of the level, E = energy of the level

Get a significant population in the upper level when

$$T \approx \frac{E_B - E_A}{k},$$

e.g., for excitation energy $\Delta E = 1$ eV

=> $T = 11{,}600$ K.

T from this definition known as the excitation temperature.

Ionization Equilibrium

When T is high enough, significant number of atoms are ionized.

Steady-state balance between ionization and recombination,

$X \leftrightarrow X^+ + e^-$, yields

$$\frac{N_+}{N_0} = \left[A(kT)^{3/2} / N_e\right] \exp(-\mathrm{X}_0 / kT) \quad \text{Saha equation}$$

N_+ = # of ions, N_0 = # of neutral atoms, N_e = # of electrons,
A = constant, X_0 = ionization potential from ground state

Boltzmann and Saha equations combined

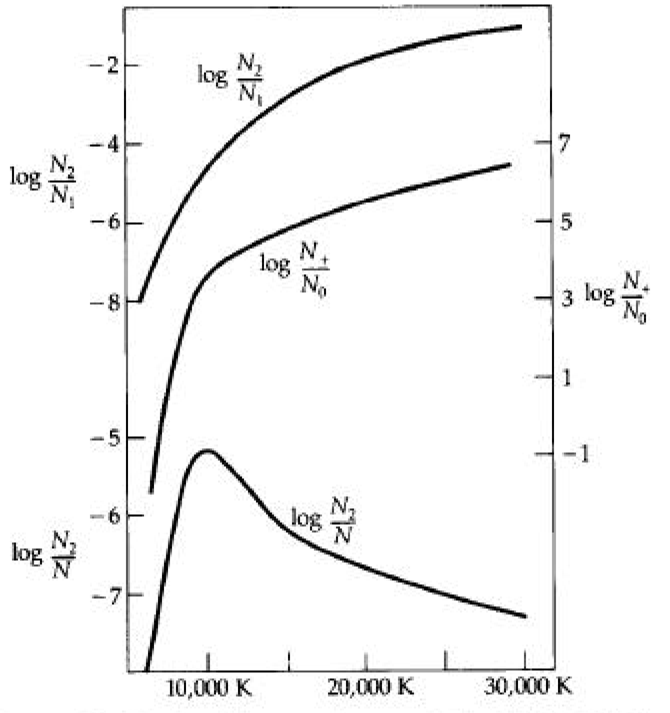
Practical problem: calculate fraction of all atoms/ions in a given state, e.g., calculate fraction of all H atoms/ions in *n*=2 state of H.

$$\frac{N_2}{N} = \frac{N_2}{N_0 + N_+} = \frac{N_2/N_0}{1 + N_+/N_0}.$$

Now, in most cases $N_0 \approx N_1$

$$\Rightarrow \frac{N_2}{N} \cong \frac{N_2/N_1}{1 + N_+/N_0}.$$

Use Boltzmann and Saha eqns =>



Note: $T < 7000$ K neutral

$T > 10000$ K ionized

N_2/N peaks at ~ 10000 K. Balmer absorption lines strongest at this temperature.

Equilibrium Properties of Radiation

Strongly interacting atoms => continuous spectrum of radiation

Planck (1900) => emission/absorption in discrete packets - “photons”. Planck’s “Blackbody” Law of Radiation applies to any sufficiently opaque body (but not really “black”), where photons are continually absorbed and re-emitted (e.g., inside a star).

$$B_l(T) = \frac{2hc^2}{l^5}\frac{1}{e^{hc/lkT}-1},$$

$$B_n(T) = \frac{2hn^3}{c^2}\frac{1}{e^{hn/kT}-1},$$

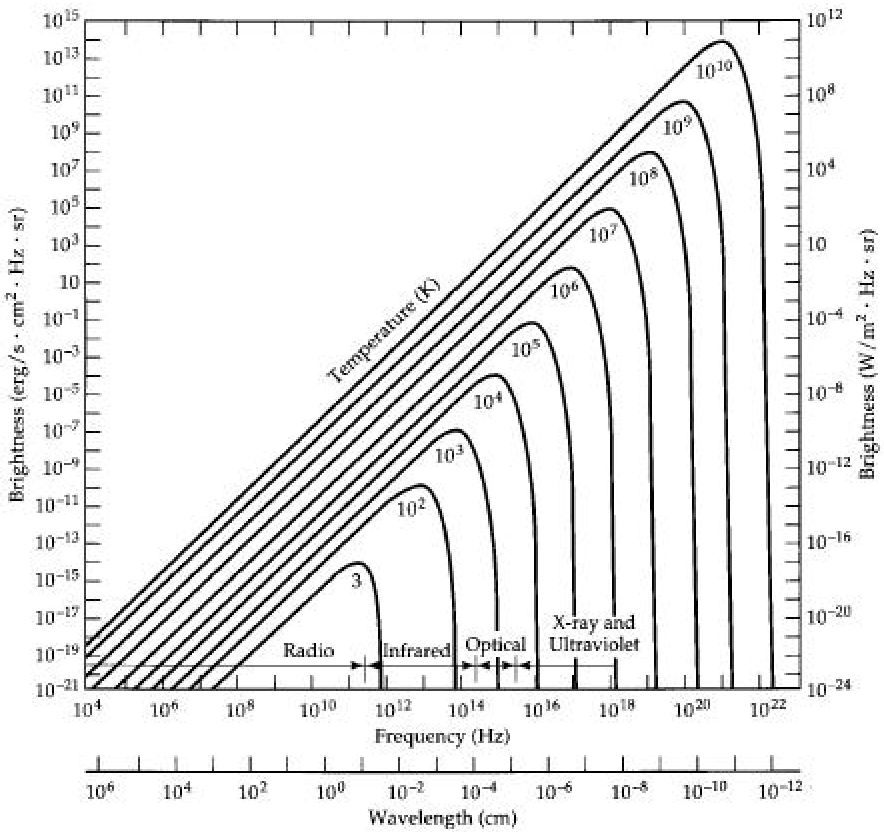
where *B* is the emitted energy per unit time per unit wavelength (or frequency) per unit area (on emitting surface) per unit solid angle (on receiver).

Planck's Blackbody Radiation Law

High frequency limit $h\nu >> kT$

$$B_{n,l}(T) = \frac{2hn^3}{c^2}\exp(-hn/kT) = \frac{2hc^2}{l^5}\exp(-hc/lkT)$$

Wien distribution



Low frequency limit $h\nu << kT$

$$B_{n,l}(T) = \frac{2n^2kT}{c^2} = \frac{2ckT}{l^4}$$

Rayleigh-Jeans distribution

Planck's Blackbody Radiation Law

Wien's Law: Peak intensity occurs at $\lambda_{max} T = 2.898 \times 10^{-3}$.

Stefan-Boltzmann Law: Total energy flux (energy/[time area]), i.e., integrated over all wavelengths and solid angles

$F = sT^4$, where $s = 5.669 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$.

Application: Total power emitted by a star, $L = 4\pi r^2 \sigma T^4$.

Summary: important qualitative properties of a BB radiator

(1) emits some energy at all wavelengths

(2) a hotter object emits a greater proportion of its energy at shorter wavelengths; peak of spectrum at shorter wavelength

(3) a hotter object emits more power at all wavelengths than a cooler one

Measuring Temperature

Matter:

kinetic temperature - from thermal Doppler broadening

excitation temperature - from Boltzmann equation

ionization temperature - from Saha equation

Radiation:

color temperature - from shape of Planck curve or Wien's Law

effective (radiation) temperature - from Stefan-Boltzmann Law

Therefore, no unique measure of temperature, but all should be equal if <u>thermodynamic equilibrium</u> holds.