Dynamics of the Earth

Time

Historically, a day is a time interval between successive upper transits of a given celestial reference point.

<u>upper transit</u> – the passage of a body across the celestial meridian moving westward

<u>hour angle</u> – the westward angular distance of an object from the meridian; negative if object is east of meridian

<u>local sidereal time</u> - hour angle of vernal equinox (fixed on CS)

<u>apparent solar time</u> - hour angle of the Sun plus 12 hr; not constant during year due to changing speed of Earth's orbit

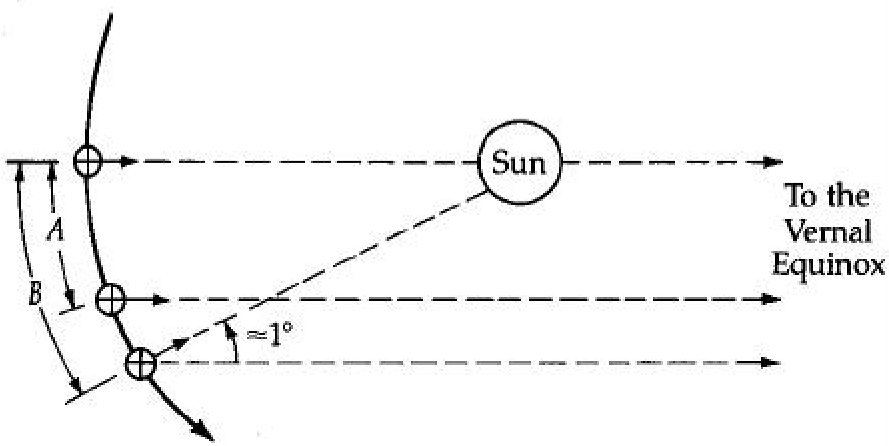
<u>mean solar time</u> - hour angle of "mean Sun" that moves along ecliptic at average angular rate of the Sun

sidereal day - two successive transits of the vernal equinox; 23 hrs, 56 min

mean solar day - two successive upper transits of mean Sun; 24 hrs

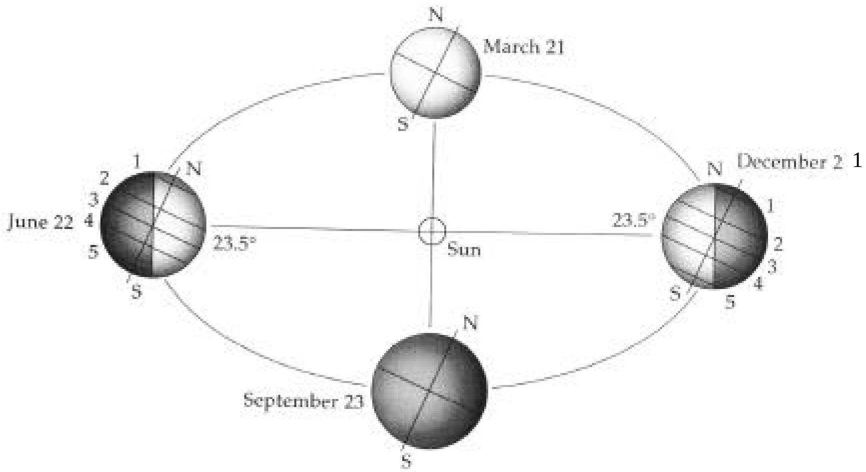
sidereal year - 365.2564 mean solar days, measured with respect to the stars

Note that stars will rise 4 min earlier every mean solar day.



The geometric origin of the difference between sidereal and solar days.

Seasons



An effect of the inclination of the Earth's equator to the ecliptic.

Note: ellipticity of Earth's orbit plays a minor role.

Equinoxes - 12 hr day and night; noontime Sun at altitude 90^0 (zenith) at equator, 0^0 at poles

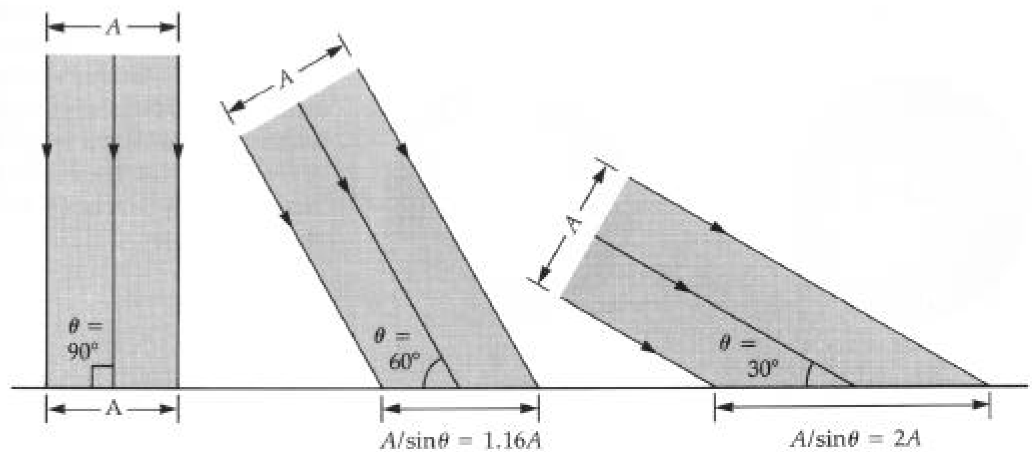
Summer solstice - Sun at highest (lowest) point in sky in the N (S) hemisphere; noontime Sun at zenith at 23.5^0 latitude (Tropic of Cancer); summer (winter) in N (S) hemisphere

Winter solstice - Sun at lowest (highest) point in N (S) hemisphere; noontime Sun at zenith at -23.5^0 latitude (Tropic of Capricorn); winter (summer) in N (S) hemisphere

Seasons

Q. Why does the Sun at a higher altitude lead to higher temperatures?

Solar insolation - the local heating effectiveness of solar energy



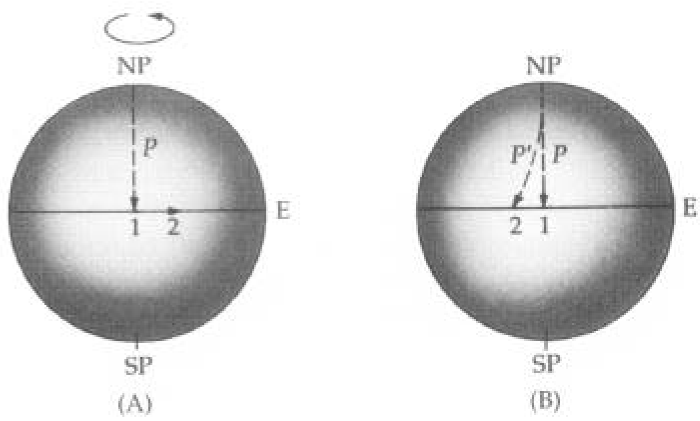
A unit of solar energy is spread over a larger area as θ decreases => heating efficiency decreases.

Q. Which months should be the hottest and coldest in the Northern hemisphere, by the above argument? Is this true?

Evidence of the Earth's rotation

The Coriolis Effect

Moving bodies in a rotating frame of reference appear to be deflected by the Coriolis acceleration. A fictitious force, like the centrifugal acceleration.



$\mathbf{a}_{\text{Coriolis}} = 2\,(\mathbf{v} \times w)$

w = angular velocity, parallel to axis

$\mathbf{v}$ = velocity

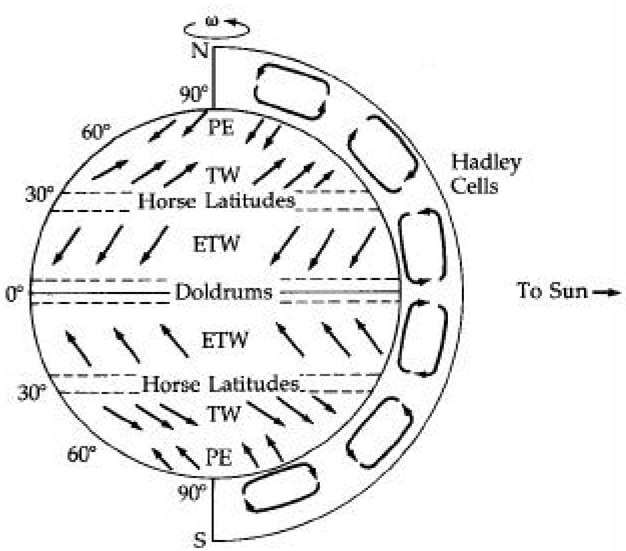
$\mathbf{a}_{\text{Coriolis}}$ is perpendicular to $\mathbf{v}$, w

Deflection is perpendicular to instantaneous velocity.

The Coriolis Effect and the Earth's Weather

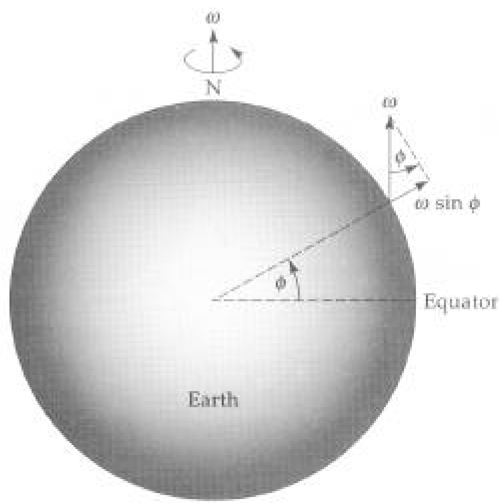
Cyclones: a flow toward a low pressure region is deflected into counterclockwise circulation in the Northern hemisphere.

Wind patterns on Earth's surface: Uneven heating of surface leads to Hadley circulation pattern in a nonrotating planet. Air rises at the warmer equator and is replaced by descending air from the colder poles.



In a rotating planet, these Hadley cells are stretched sideways by the Coriolis force. Leads to multiple Hadley cells, with alternating <u>easterly</u> and <u>westerly</u> winds.

Foucault's Pendulum



The plane of a pendulum's swing appears to rotate – Foucault (1851). A westward deflection at the North pole. An example of the Coriolis effect.

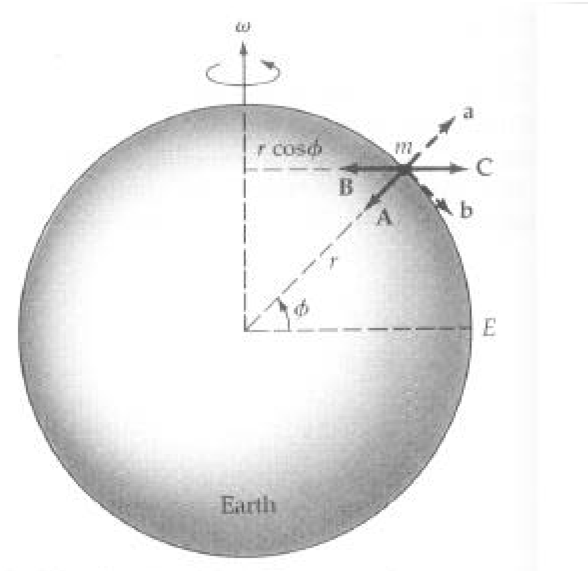
In the nonrotating frame, the Earth rotates below a pendulum at the North pole with angular speed $\omega = 2\pi/P$, where P = sidereal day.

At latitude ϕ, the vertical component of the Earth's angular speed is $\omega' = \omega \sin \phi$, so the period of the rotation of the pendulum plane is $P' = 2\pi/\omega' = 2\pi/(\omega \sin \phi)$.

Oblate Earth

Rotation changes shape of the Earth: spherical => oblate.

From a nonrotating frame of reference:



Consider a mass element at surface that is unaffected by any intermolecular forces.

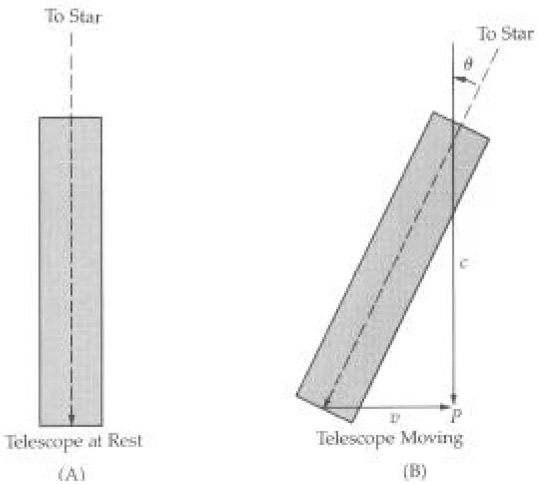
Centripetal acceleration

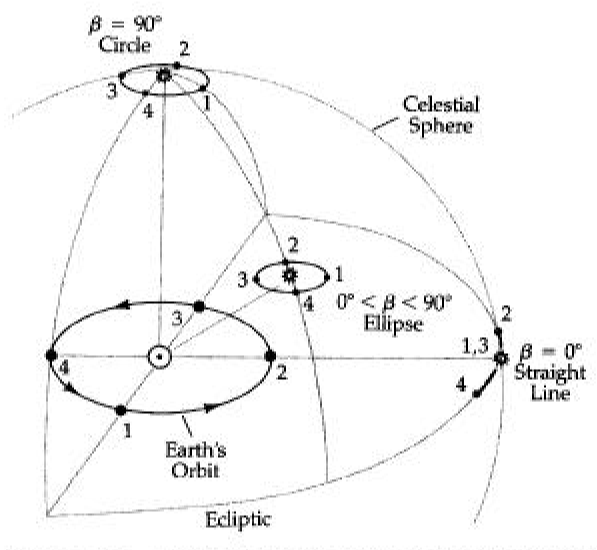
$$\mathbf{a} = -w^2 r\cos^2 f\,\hat{\mathbf{r}} + w^2 r\cos f\,\sin f\,\hat{f}$$

Gravity can provide the required radial acceleration but not the ϕ–component => results in material sliding downward along the ϕ–direction => leads to oblateness.

Evidence of the Earth's revolution about the Sun

Aberration of starlight





Telescope moving at speed *v* must be tilted so that light reaches bottom.

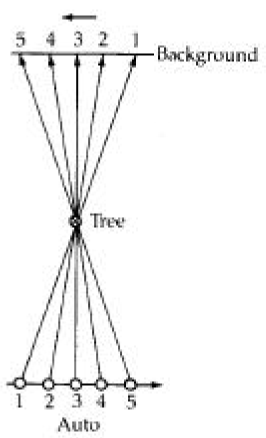
$$\Delta t = \frac{L \sin q}{v} = \frac{L \cos q}{c} \Rightarrow \frac{v}{c} = \tan q \approx q \,.$$

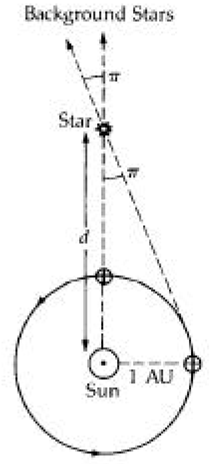
The apparent position of a star oscillates as the Earth moves around the Sun. Path depends on the star's direction, and not its distance.

James Bradley (1729) measured angular radius θ = 20.49" => *v* = 29.8 km/s. Compare to *c* = 3 x 10^5 km/s.

Stellar Parallax

Nearby stars appear to move with respect to the background as the Earth moves in its orbit.





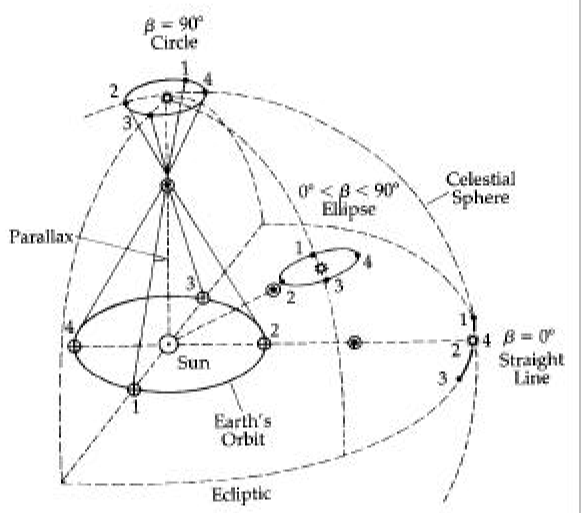
This effect does depend on distance.

$$\tan q = \frac{1}{d} \Rightarrow d = \frac{1}{\tan q} \approx \frac{1}{q} \quad \text{AU},$$

where θ is in radians. Can rewrite as

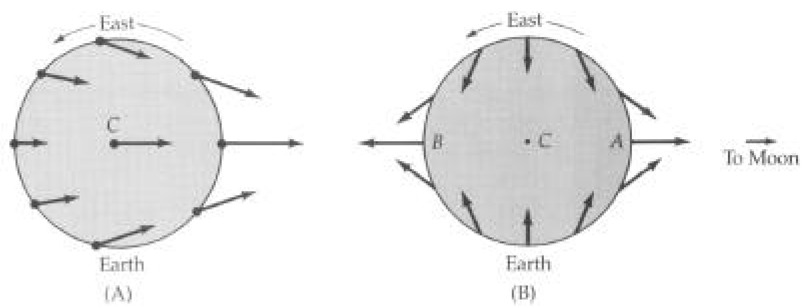
$$d = \frac{206{,}265}{p''}\,\text{AU} = \frac{1}{p''}\ \text{pc},$$

where π is in arcseconds. This defines the parsec (pc).



The angular size of the parallactic orbit depends on distance to the star. This effect is 90° out of phase with aberration orbit.

Tides



Forces Differential Forces

Tides occur due to differential gravitational force.

$$F = \frac{GMm}{r^2} \Rightarrow dF = \left(\frac{-2GMm}{r^3}\right) dr,$$

Q. Which has a greater tidal effect, the Sun or the Moon?

$$\frac{F_{Sun}}{F_{Moon}} = \left(\frac{M_{Sun}}{M_{Moon}}\right)\left(\frac{r_{Moon}}{r_{Sun}}\right)^2 \approx 177, \text{ but}$$

$$\frac{dF_{Sun}}{dF_{Moon}} = \left(\frac{M_{Sun}}{M_{Moon}}\right)\left(\frac{r_{Moon}}{r_{Sun}}\right)^3 \approx 5/11.$$

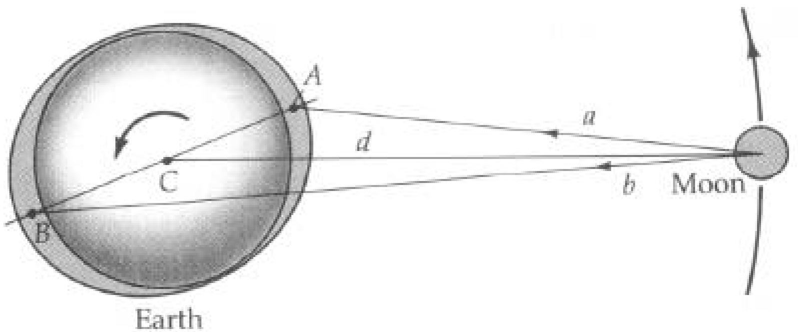
Moon at conjunction or opposition => spring tides.

Moon at quadrature => neap tides.

Tidal Friction

Tide raising forces => internal tidal friction =>

tidal evolution and synchronous rotation .



Can show that

$$L \propto dv \propto d^{1/2},$$

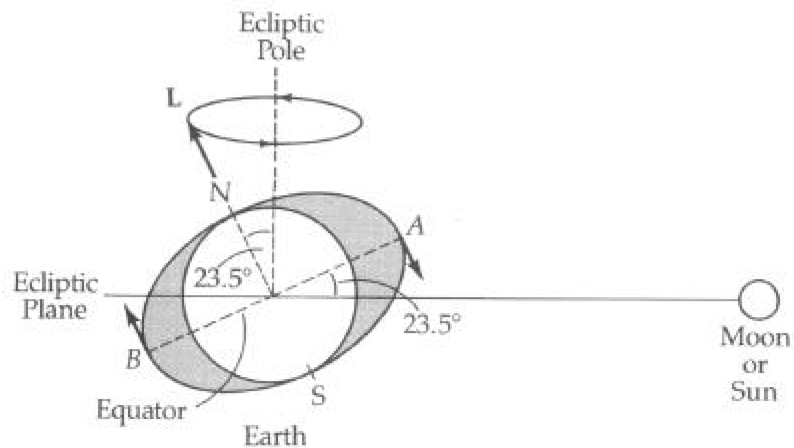
using Kepler's 3rd Law.

Earth and Moon lose rotational energy and evolve towards synchronous rotation, where rotation period = orbit period (this has already happened to the Moon). Total angular momentum is conserved; conversion from spin to orbital angular momentum, which increases semimajor axis of orbit.

Q. Why does synchronous rotation minimize internal friction?

Precession and Nutation

Effect of tidal forces on Earth's equatorial bulge.



$t = d\mathbf{L}/dt = \mathbf{r} \times \mathbf{F}$

Differential of forces acting at bulges makes the spinning Earth's axis precess about ecliptic pole. Analogy to a spinning top.

Consequence: Celestial coordinates keep changing. The celestial pole traces a circular path with period 26,000 yr => precession of equinoxes 360^{o}/26,000 yr = 50"/yr along ecliptic. Celestial coordinates must be updated to current epoch.

Also, motion of Sun/Moon above and below equatorial plane => nutation, or wobbling, of the Earth's rotation axis. 9'' amplitude and 18.6 yr period.

Roche Limit

Tidal forces tend to tear a satellite apart. This occurs when the distance between the center of a satellite and primary is less than

$$d = k\left(\frac{r_M}{r_m}\right)^{1/3} R,$$

where ρ_M and R are the density and radius of the primary, and ρ_m is the density of the satellite.

$k = 2.44$ for a fluid,

varies slightly for materials of different composition.

Rings of Saturn and other giant planets all lie within their Roche limit.