Celestial Mechanics

The Heliocentric Model of Copernicus

Sun at the center and planets (including Earth) orbiting along circles.

inferior planets - planets closer to Sun than Earth - Mercury, Venus

superior planets - planets farther from Sun than Earth - all other planets

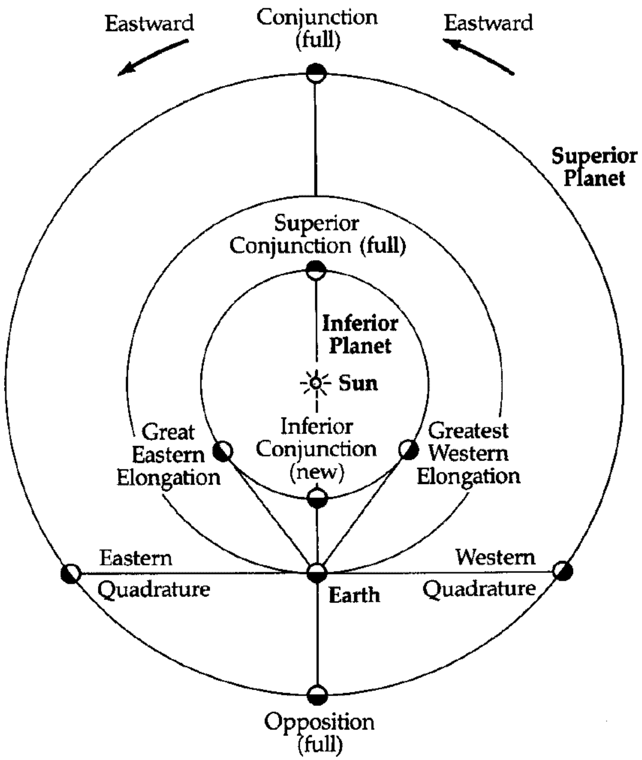
elongation - the angle seen at the Earth between the direction to the Sun's center and the direction to the planet

Assumed that outer planets revolve more slowly than inner ones.

This model provides elegant explanations for observed elongations, retrograde motion, and phases of the planets.

No better at fitting contemporary data than refined Ptolemaic model.

Heliocentric Model



<u>conjunction</u> - an elongation of 0^0

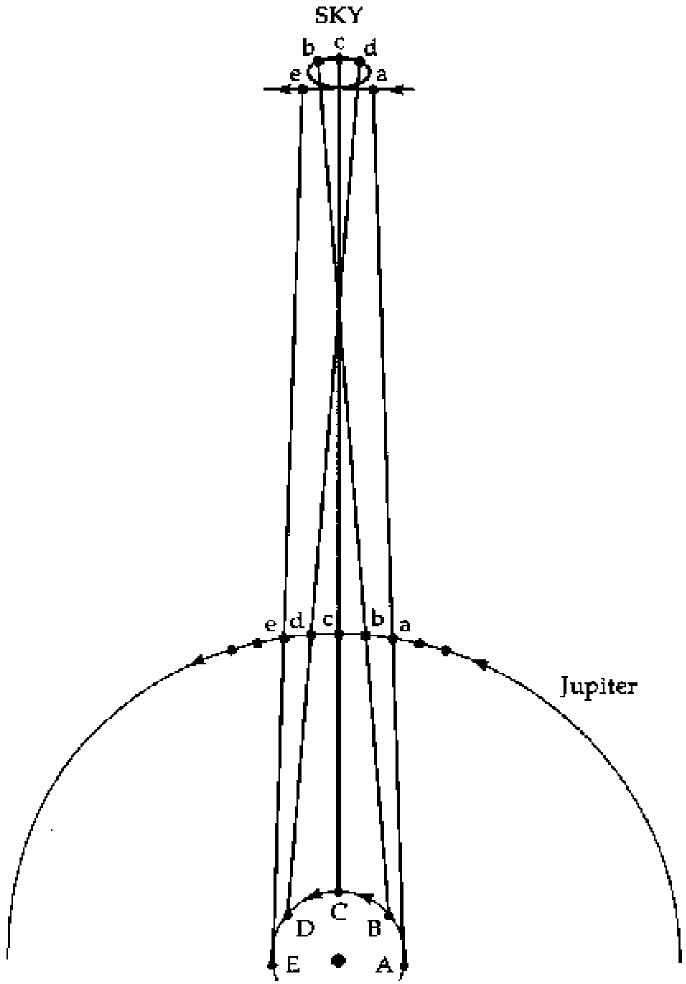
<u>opposition</u> - elongation of 180^0; only superior planets

<u>quadrature</u> - elongation of 90^0; only superior planets

<u>greatest elongation</u> - only for inferior planets

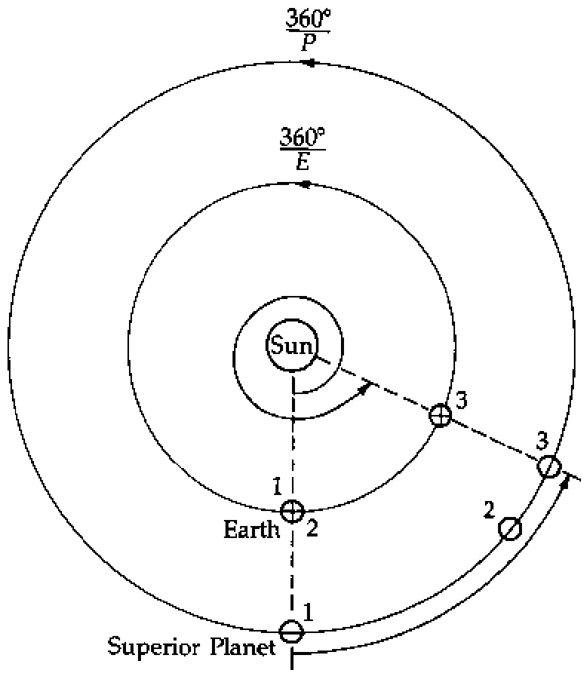
Model naturally explains occurrence of opposition, quadrature, and greatest elongation of various planets. Also existence of all phases of inferior planets - later verified by Galileo for Venus. Finally, it provides a simple explanation for retrograde motion.

Retrograde motion



The faster inner planet moves past the slower outer planet.

Synodic and Sidereal Periods



synodic period = S = time for a planet to return to the same position in the sky relative to the Sun, as seen from Earth

sidereal period = P = time for a planet to complete one complete orbit of the Sun relative to the stars

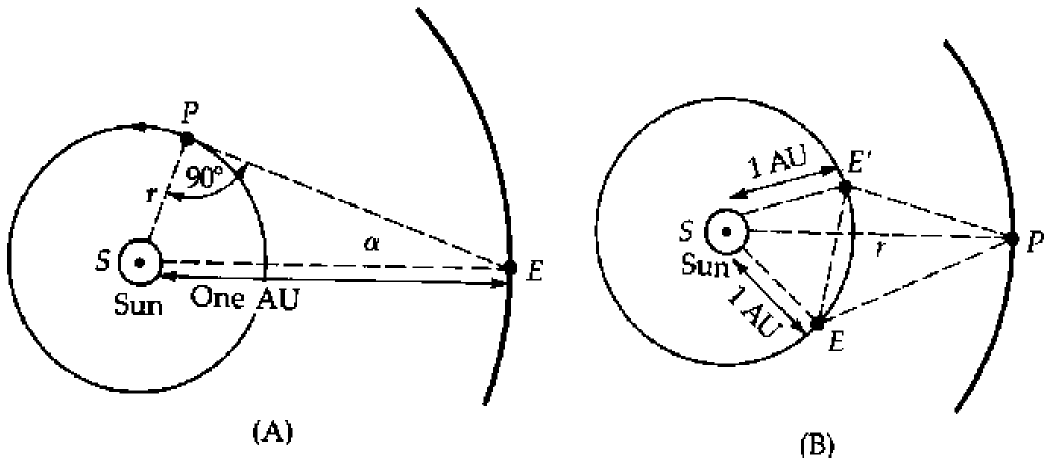
$$\frac{1}{S} = \frac{1}{P_A} - \frac{1}{P_B},$$

where P_A = sidereal period of inferior planet, and

P_B = sidereal period of superior planet.

Kepler's Methods

Triangulation - to find each planet's distance from the Sun.



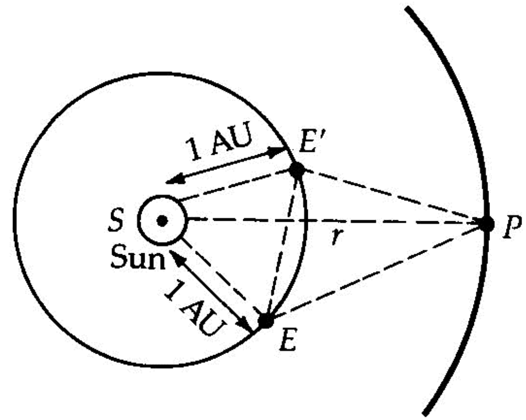
(A) Inferior planet - $r = \sin \alpha$, in Astronomical Units (AU), where α = greatest elongation angle

(B) Superior planet - Determine sidereal period *P*, then measure elongation angles *PES* and *PE'S* an interval *P* apart. Use trigonometric relations (including law of cosines and law of sines - see Appendix 9) to determine *r*. Determining *r* at different points on the orbit traces out the shape of the orbit.

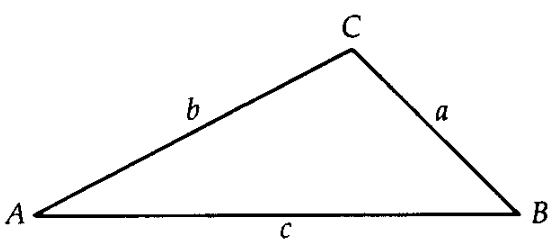
Kepler's Methods

Triangulation for a superior planet.

Use law of sines and law of cosines.



(B)



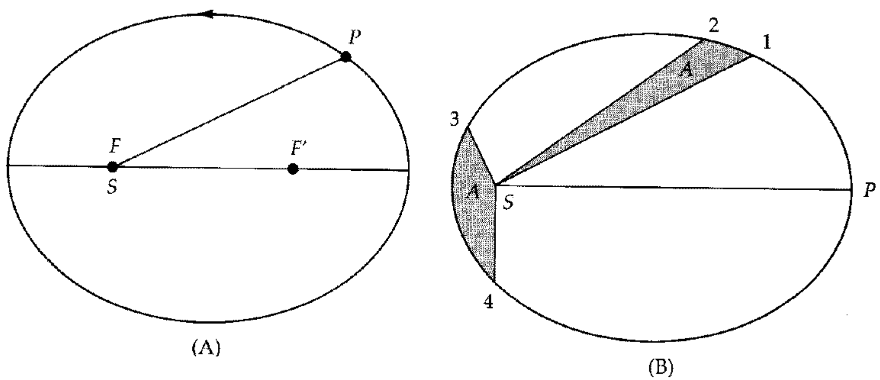
Law of sines $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines $$a^2 = b^2 + c^2 - 2bc \cos A$$

Can you list the required steps to determine *r*?

Kepler's Laws of Planetary Motion

Kepler traced out the orbits of the planets, and combined with temporal information, arrived at three empirical laws of motion.

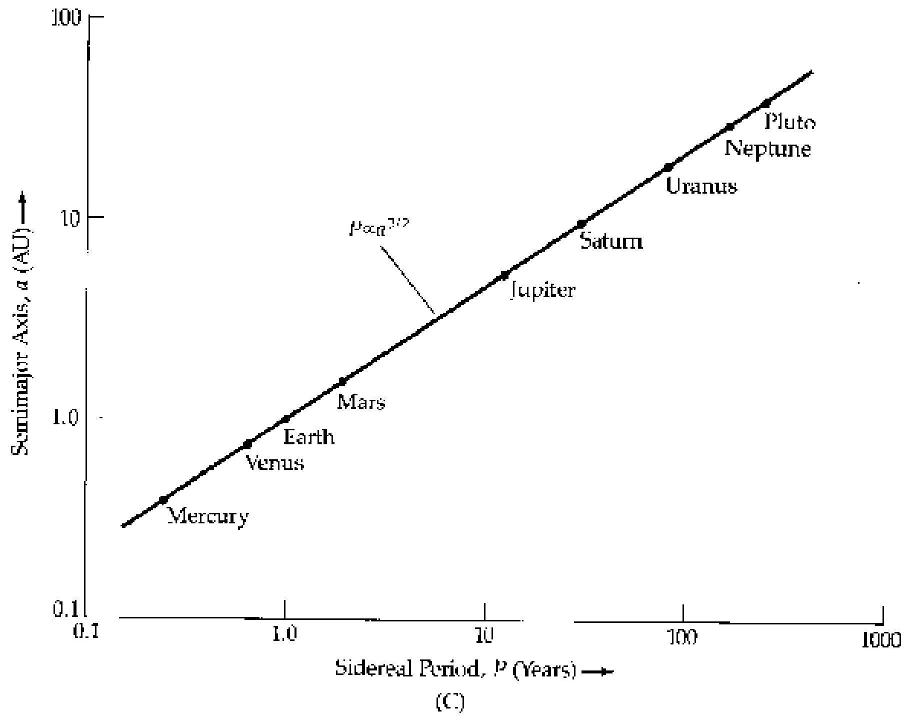


(1) Each planet traces an elliptical orbit E around the Sun (S) which is at one focus (F) of an ellipse.

(2) The radius vector to the planet sweeps out equal areas of the ellipse in equal time intervals.

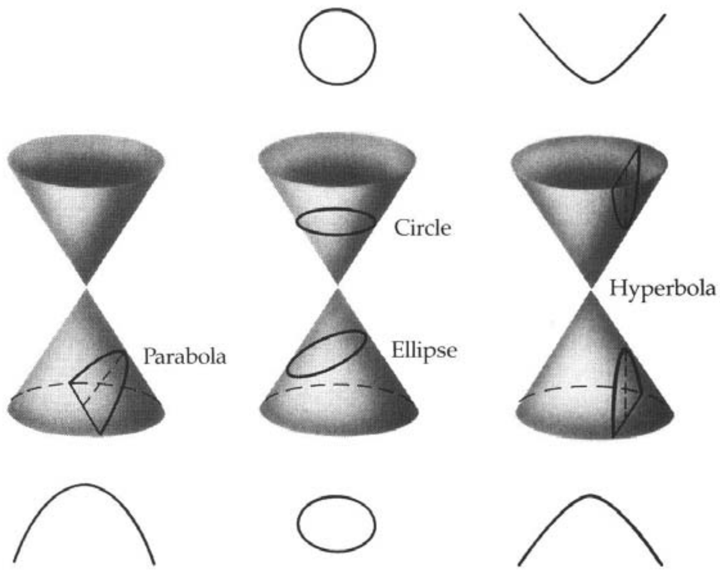
Kepler's Laws of Planetary Motion

(3) The squares of the sidereal periods of the planets are proportional to the cubes of the semimajor axes (mean radii) of their orbits => $P^2 = k\,a^3$.



Conic Sections

Curves obtained by slicing a cone at different angles.



Cut perpendicular to axis => circle

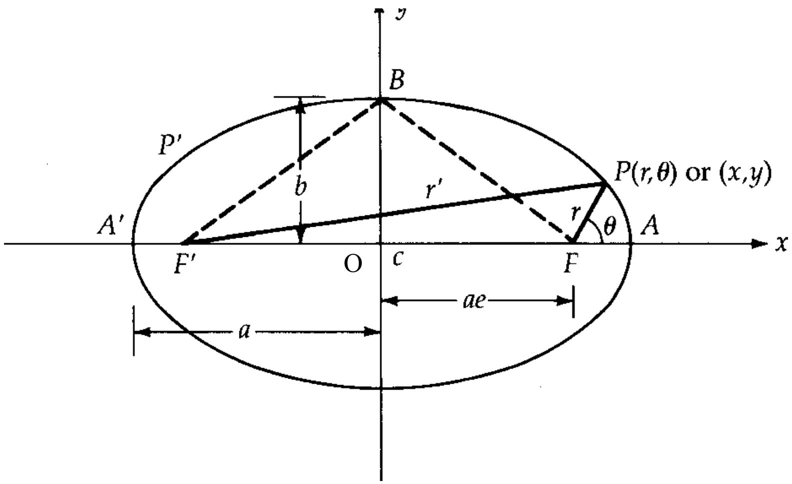
Cut parallel to side => parabola

Cut at intermediate angle to above directions => ellipse

Cut at angle greater than cone opening angle => hyperbola

Conic sections

Ellipse - the locus of all points such that the sum of distances from two foci to any point on the ellipse is constant.



$r + r' = 2a$ = constant

a = semimajor axis

b = semiminor axis

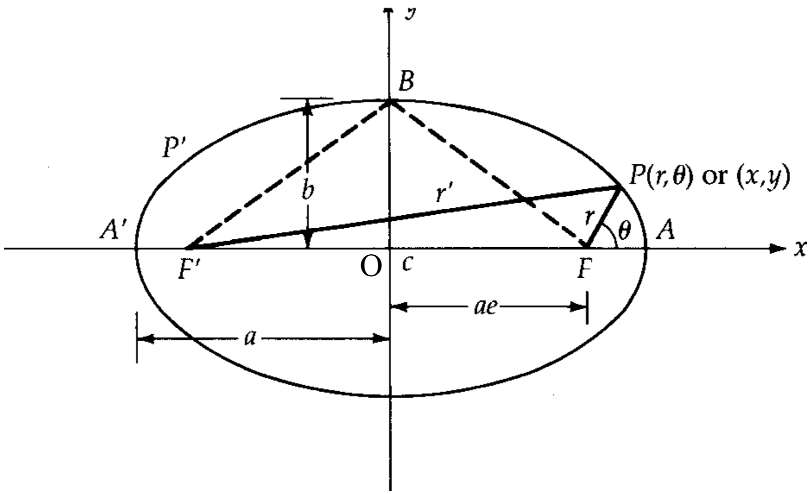
e = OF/a = eccentricity

FA = $a\,(1-e)$, FA’ = $a\,(1+e)$

$b = a\,(1-e^2)^{1/2}$

area $A = \pi\, a\, b = \pi\, a^2\, (1 - e^2)^{1/2}$

Conic Sections



Kepler's 1st Law places Sun at one focus F.

Perihelion (perigee) at A.

Aphelion (apogee) at A'.

Mean distance from F = a.

in Cartesian coordinates from origin O $$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

in polar coordinates from F $$r = \frac{a\left(1 - e^2\right)}{1 + e\cos\theta}$$

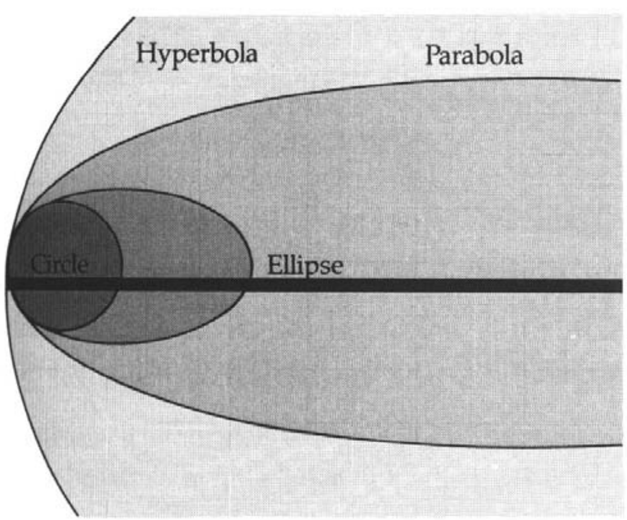
Conic sections

Limiting cases:

if $e = 0$, circle $\qquad r = a$

if $e = 1$, parabola $\qquad r = \frac{2p}{1+\cos\theta}$ $\qquad$ where p = distance of closest approach to single focus.

if $e > 1$, hyperbola $\qquad r = \frac{a\left(e^2 - 1\right)}{1 + e\cos\theta}$



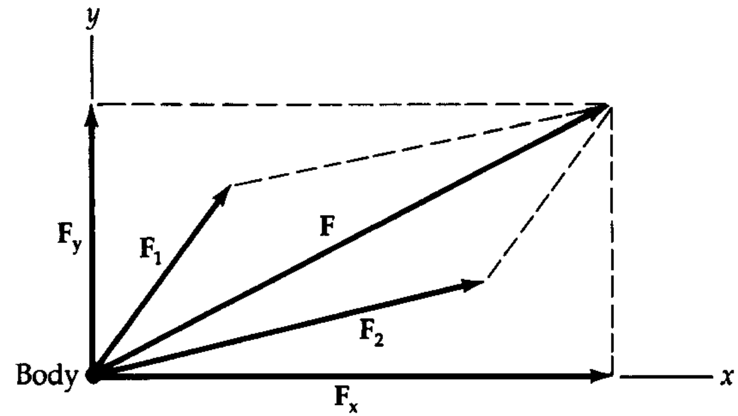
Newton's mechanics =>

relative orbit of a moving body (due to gravitational influence) must be a conic section.

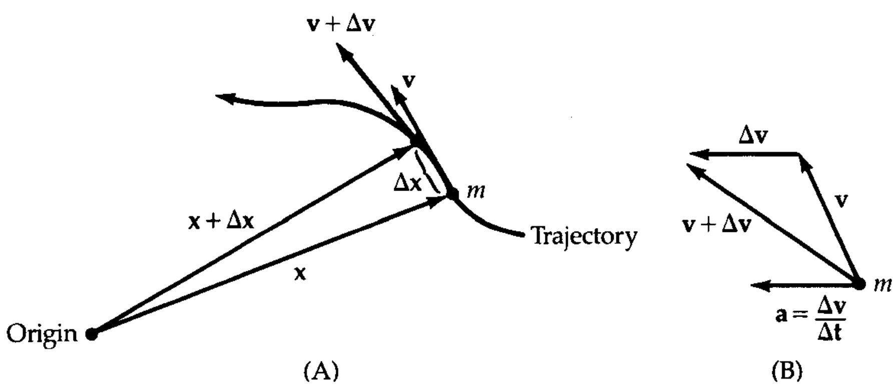
Newtonian Mechanics

Expressed in terms of vectors

Vector addition and subtraction



Position, velocity, and acceleration



$\mathbf{v} = d\mathbf{x}/dt$, $\mathbf{a} = d\mathbf{v}/dt$

Newton's Laws of Motion

First Law: $\mathbf{p} = m\,\mathbf{v} = \text{constant}$

The linear momentum of an object remains constant unless acted upon by an outside force.

Second Law: $\mathbf{F} = d\mathbf{p}/dt = m\,d\mathbf{v}/dt = m\,\mathbf{a}$

The force on a body equals the time rate of change of momentum.

Third Law: $\mathbf{F_e} = -\mathbf{F_a}$, or $\mathbf{p} = \text{constant}$ for a closed system

The force exerted by a body is equal and opposite to the force acting on the body. A consequence of the 1st and 2nd Laws.