Binary Stars

- Pairs of stars which move due to mutual gravitational attraction; orbit common center of mass
- ~2/3 of all stars are part of a binary or multiple system
- Crucial for determining stellar masses; also can yield information on radii, density, temperature, luminosity, rotation rate
- Reveal the importance of angular momentum in the star formation process

Classification of Binaries

Visual binary - two stars resolved as separate points that orbit one another.

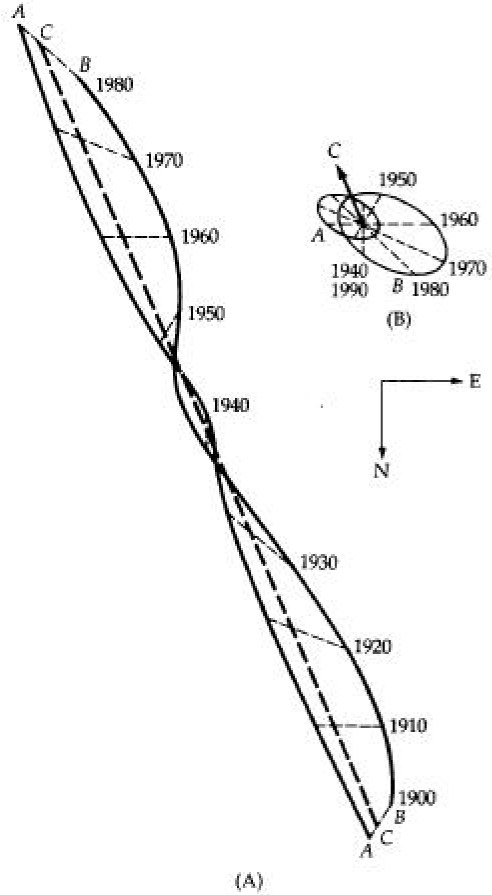
Astrometric binary - only one star is seen, but it wobbles in the sky, implying the presence of an unseen companion

Spectroscopic binary - an unresolved pair for which we see a periodic variation of the Doppler shift of spectral lines; can be double-lined, if both spectra observed, or single-lined, if only one spectrum observed

Eclipsing binary - periodic changes in the total light due to the stars eclipsing each other

Visual Binaries

Earth’s atmosphere limits resolution to $\Delta\theta < 1''$, so see binaries with large separation a => long P.



Observations of the motions of Sirius A and B, before and after center of mass motion (dashed line) is subtracted out.

Visual Binaries - Analysis

Observe apparent (projected) elliptical orbit.

Measure a in arcseconds (a''), P in yrs.

If know distance d, then a (AU) $= a'' \, d$ (pc)

Apply Kepler’s 3rd Law, $P^2 = 4\pi^2 a^3/G(m_1+m_2)$.

If also see motion of each star relative to center of mass,

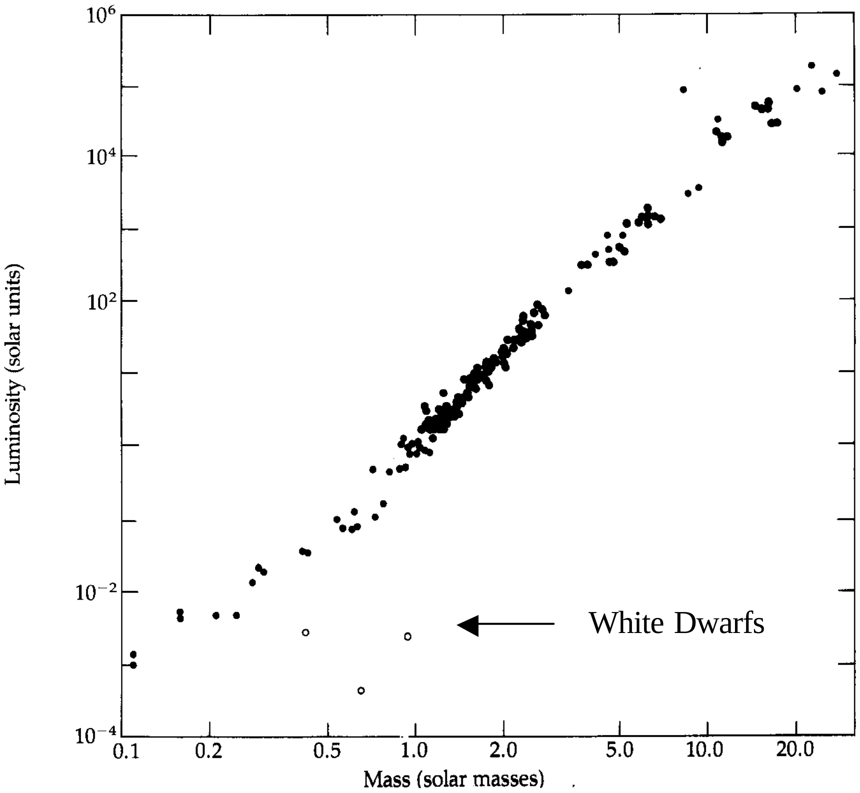
$$\frac{m_1}{m_2} = \frac{r_2}{r_1}$$

=> get m_1 and m_2.

$$m_1 + m_2 = \frac{4p^2}{G}\frac{a^3}{P^2}$$

Mass-Luminosity Relationship

Determined from binary systems in which reliable mass values can be inferred.



Note that

$10^{-1} \leq M/M_{Sun} \leq 10^2$, but

$10^{-4} \leq L/L_{Sun} \leq 10^6$!

Approximate relations

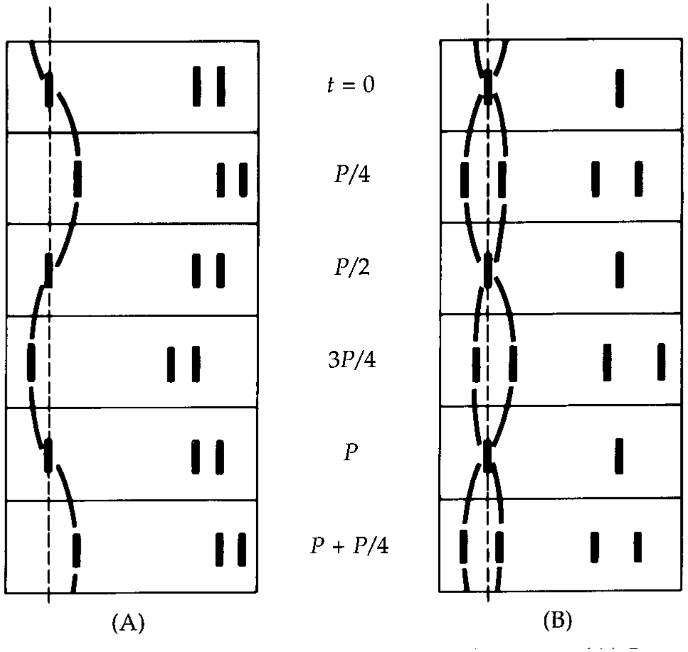
$L \propto M^a$, where $\overline{a} = 3.3$.

More specifically,

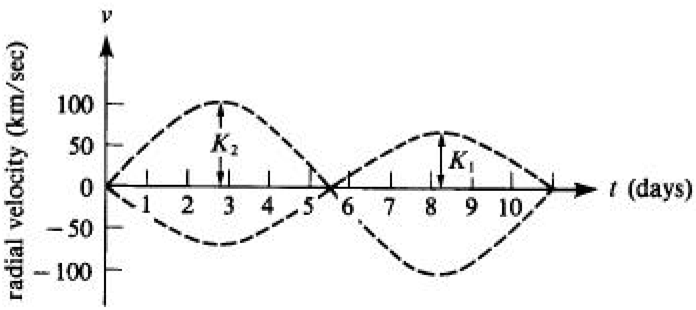
$a = 2.3$ for $M < 0.43 M_{Sun}$,

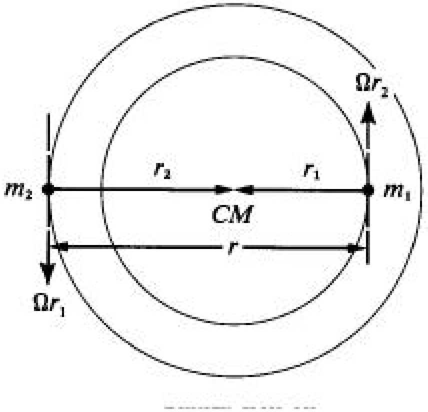
$a = 4.0$ for $M > 0.43 M_{Sun}$.

Spectroscopic Binaries



Single-lined Double-lined





Measure $K_1 = v_1 \sin i$ and sometimes $K_2 = v_2 \sin i$, where i is the inclination angle between orbit plane and the plane of the sky.

Spectroscopic Binaries - Analysis

Apply laws of orbital motion, using measured K_1, K_2, and P. Look at three cases:

(1) Double-lined, $i = 90^0$ (edge-on), circular orbit <=> sinusoidal velocity curve: we observe P, v_1, v_2.

$$P = \frac{2p\, r_1}{v_1} = \frac{2p\, r_2}{v_2} \quad \Rightarrow \quad r_1 = \frac{Pv_1}{2p}, \quad r_2 = \frac{Pv_2}{2p}$$

$$\Rightarrow \quad a = r_1 + r_2 = \frac{P}{2p}\left(v_1 + v_2\right).$$

This leads to

$$\frac{m_2}{m_1} = \frac{r_1}{r_2} = \frac{v_1}{v_2},$$

=> solve to get m_1 and m_2.

$$m_1 + m_2 = \frac{4p^2}{G}\frac{a^3}{P^2} = \frac{P}{2p\, G}\left(v_1 + v_2\right)^3.$$

Spectroscopic Binaries - Analysis

(2) Double-lined, arbitrary i, circular orbit: observe P, $K_1 = v_1 \sin i$, $K_2 = v_2 \sin i$.

$$\frac{m_2}{m_1} = \frac{r_1}{r_2} = \frac{v_1}{v_2} = \frac{K_1}{K_2} \quad (1) \quad (\text{note} : \sin i \text{ cancels out})$$

Also:

$$P = \frac{2\pi r_1}{v_1} = \frac{2\pi r_1 \sin i}{K_1} = \frac{2\pi r_2 \sin i}{K_2} \quad \Rightarrow r_1 = \frac{K_1 P}{2\pi \sin i}, \quad r_2 = \frac{K_2 P}{2\pi \sin i}$$

$$\Rightarrow \quad a = r_1 + r_2 = \frac{P}{2\pi \sin i}\left(K_1 + K_2\right).$$

$$m_1 + m_2 = \frac{4\pi^2}{G}\frac{a^3}{P^2} = \frac{P}{2\pi G}\frac{\left(K_1 + K_2\right)^3}{\sin^3 i} \quad \Rightarrow \left(m_1 + m_2\right)\sin^3 i = \frac{P}{2\pi G}\left(K_1 + K_2\right)^3 \quad (2)$$

Solve equations (1) and (2) for $m_1 \sin^3 i$ and $m_2 \sin^3 i$.

Spectroscopic Binaries - Analysis

(3) Single-lined, arbitrary i, circular orbit: observe P, $K_1 = v_1 \sin i$.

$$r_1 = \frac{v_1 P}{2\pi} = \frac{K_1 P}{2\pi \sin i}.$$

$$a = r_1 + r_2 = r_1 \left(1 + m_1/m_2\right) = \frac{K_1 P}{2\pi \sin i}\left(1 + m_1/m_2\right).$$

Also,

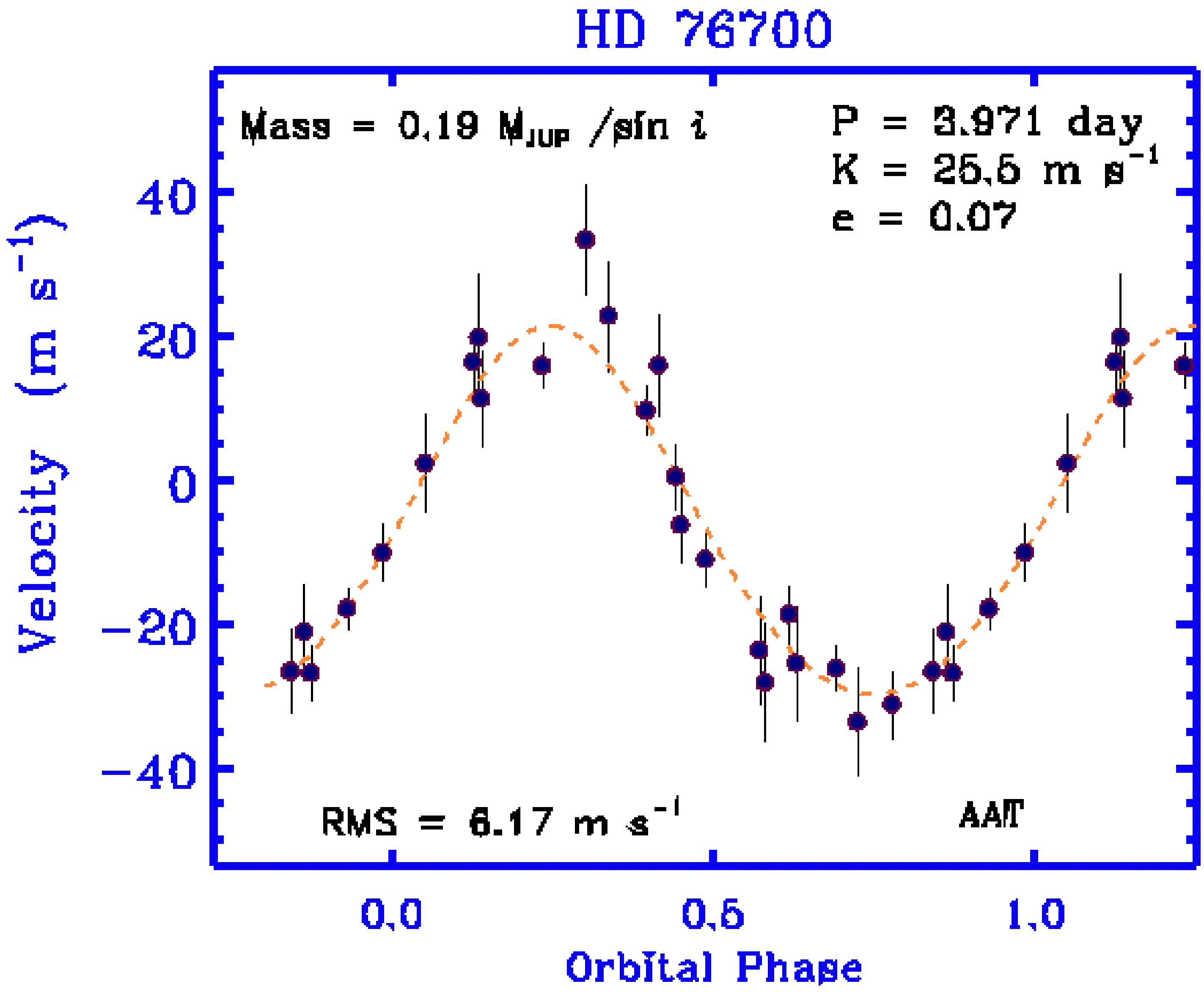
$$m_1 + m_2 = \frac{4\pi^2}{G}\frac{a^3}{P^2} = \frac{K_1^3 P}{2\pi G \sin^3 i}\left(1 + m_1/m_2\right)^3 \quad \Rightarrow \quad m_2^3 \sin^3 i = \frac{K_1^3 P}{2\pi G}\left(m_1 + m_2\right)^2.$$

If $m_1 \gg m_2$ and can estimate m_1, then

$$m_2 \sin i = \frac{K_1 P^{1/3}}{\left(2\pi G\right)^{1/3}} m_1^{2/3}.$$

Yields a lower limit to m_2, the mass of the unseen companion. This method used to find planets too.

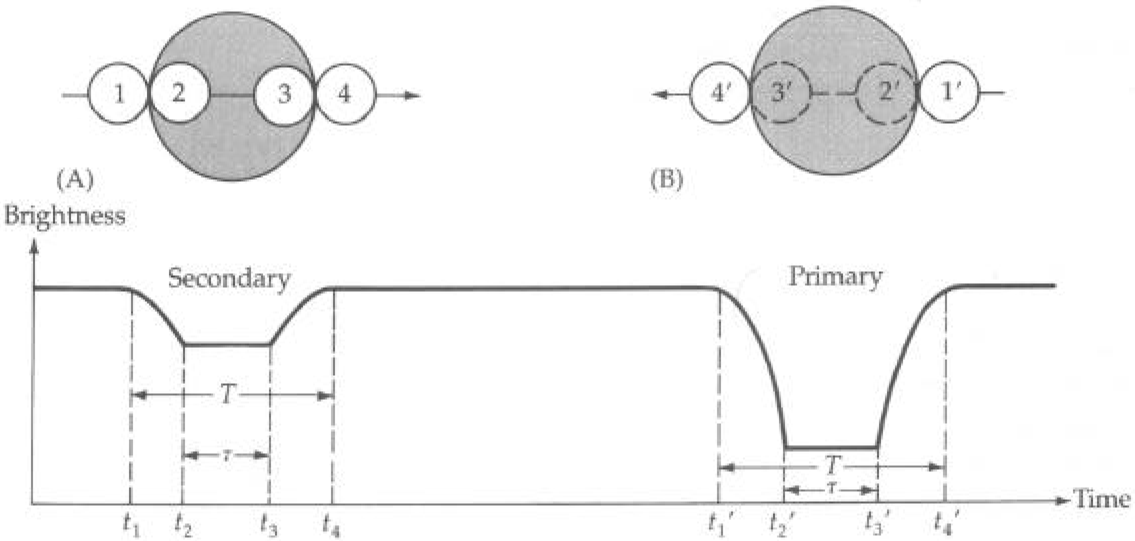
Planet Detection using Spectroscopic Method



Tinney et al.
(2002)

Eclipsing Binaries

$i = 90^0$, circular orbit



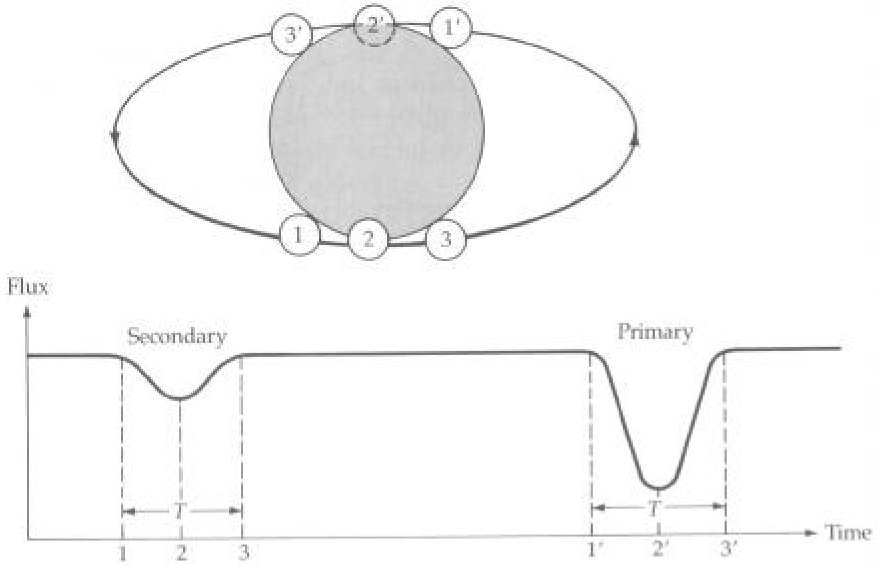
$$2R_s = v(t_2 - t_1) = v(t_4 - t_3),$$

$$2\left(R_s + R_l\right) = v(t_4 - t_1).$$

Also $a = \dfrac{vP}{2p} \Rightarrow \dfrac{R_s}{a} = p\,\dfrac{t_2 - t_1}{P}, \dfrac{R_l}{a} = p\,\dfrac{t_4 - t_2}{P}.$

Eclipsing Binaries

$i \neq 90^0$, but not very far off.



Can determine i from shape of light curve.

Note that an eclipsing binary (EB) is also likely a spectroscopic binary (SB).

Also, note that $$\frac{\text{depth of primary}}{\text{depth of secondary}} = \frac{F_{\text{primary}} \times \text{area eclipsed}}{F_{\text{secondary}} \times \text{area eclipsed}} = \left(\frac{T_{\text{hotter}}}{T_{\text{cooler}}}\right)^4.$$

Complementarity of eclipsing (EB) and spectroscopic (SB) data:

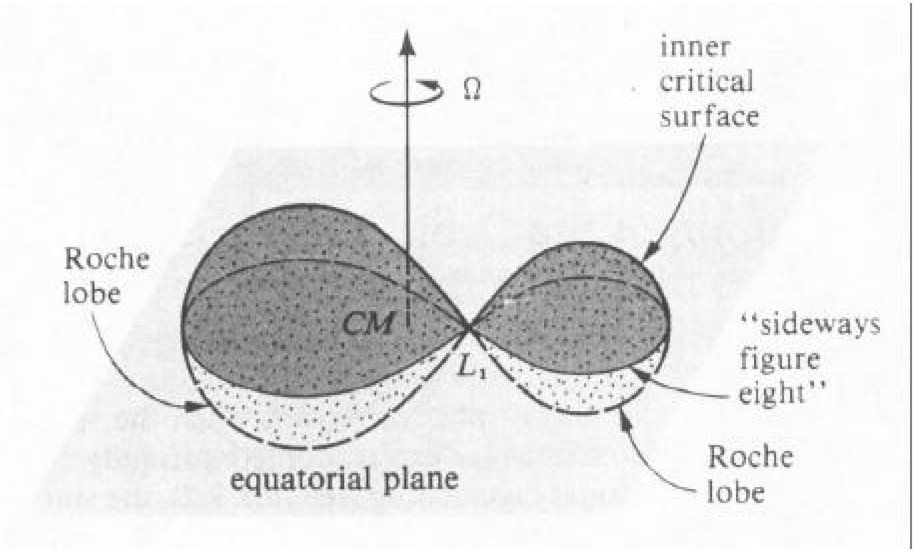
EB => R_s/a, R_l/a, i

=> get m_1, m_2, a, R_l, R_s!

SB => $a \sin i$, $m_1 \sin^3 i$, $m_2 \sin^3 i$

Close Binaries

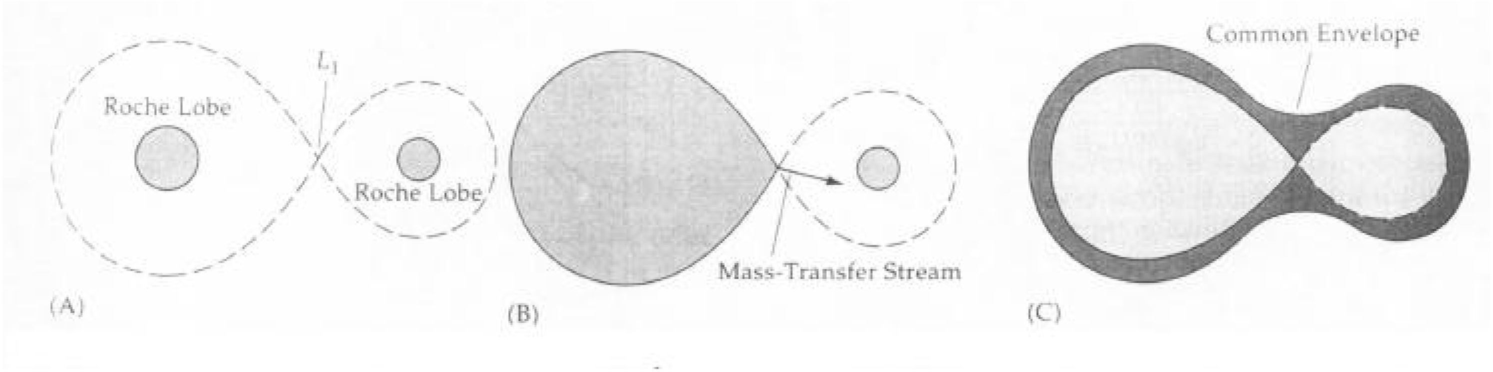
Consider a binary system with short separation *a* and period *P*.



Transform to a frame where the stars are stationary, i.e., a frame rotating at a rate $\Omega = 2\pi/P$. In this frame, $g_{\text{eff}} = g_1 + g_2 - \Omega^2 r$, where r is the distance to the CM, g_1 and g_2 are the gravitational fields of the two stars.

A sideways figure-eight defines the two regions where g_1 and g_2 dominate, respectively. These regions are called the Roche lobes.

Close Binaries



Three broad categories:

(A) detached system - Both stars are smaller than their Roche lobes

(B) semi-detached system - One star fills its Roche lobe and mass flows to the companion.

(C) contact system - Both stars fill their Roche lobes. The system is shrouded by a common envelope of material.