ASTRONOMY 020

Problem Set #4

Due: October 10, 2003

1. We showed in class that Newton used his laws of motion and insight into circular motion to determine that
$$F_{\rm grav} = \frac{4\pi^2 m}{kr^2}, \tag{1}$$
where m is the mass of the object undergoing a circular path, r is the radius of the path, and k is the constant in Kepler's Third Law, $P^2 = kr^3$, relating the period P and r. Newton further postulated that the gravitational force exerted by the Earth on a satellite has magnitude
$$F_{\rm grav} = \frac{GM_\oplus m}{r^2}, \tag{2}$$
where G is the gravitational constant, $M_\oplus$ is the Earth's mass, and m is the mass of the satellite in an orbit of radius r.

   Use the data for the Moon's orbit around the Earth (see Appendix 3 of your textbook) to find a numerical value for $GM_\oplus$. Discuss why Newton could not determine G or $M_\oplus$ separately.

   Write a paragraph explaining how G (and therefore also $M_\oplus$) was finally determined.

2. Zeilik & Gregory, Chapter 1, problem 7.

3. An artificial satellite is to be launched into orbit around the Earth. Suppose the launch point is $2.5R_\oplus$ from the center of the Earth and the launch direction is perpendicular to the line joining the launch point to the Earth's center.

   (a) What is the speed $v_{\rm c}$ required to go into circular orbit?

   (b) What is the minimum speed to escape the Earth's gravitational field?

   In the following parts, assume the satellite is launched with speed $1.2v_{\rm c}$.

   (c) What is the semimajor axis a of the orbit?

   (d) What is the orbital eccentricity?

   (e) What is the orbit period?

Practice problems:

1. Zeilik & Gregory, Chapter 1, problem 6.
   Answers: (a) $r_{\rm Sun-CM} = 1.07R_\odot$, (b) $r_{\rm E-CM} = 0.74R_\oplus$, thus within the Earth.

2. Zeilik & Gregory, Chapter 2, problem 19.
   Answer: The escape speed from Earth's surface is $v_{\rm esc} = 11$ km/s. The escape speed from the Sun at a distance of 1 AU is $v_{\rm esc} = 42$ km/s. However, the speed of the Earth in its orbit around the Sun has an average value $v = 30$ km/s, so a satellite launched in the right direction needs only an additional 12 km/s relative to the Earth to escape the Sun.