ASTRONOMY 020

Practice Set #23
April 2, 2004
Not to be handed in

1. Use the equation for energy conservation for an expanding universe in a Newtonian cosmology and Hubble's law to derive the critical density for the universe.
   Answer: $\rho_{\text{crit}} = 3H^2/(8\pi G)$, where $H = \dot{R}/R$.

2. Obtain a rough estimate for the mass density in the Local Group (mostly the mass of the Milky Way and M31). Assume a total mass $4 \times 10^{11}\ M_\odot$ and a cubic volume with sides 4.6 Mpc. Compare this number to the critical density of the universe if $H_0 = 65$ km s^{-1} Mpc^{-1}.
   Answer: $\Omega = \rho/\rho_{\text{crit}} \approx 0.035$, not nearly enough to close the universe.

3. For a flat universe, show that
   $$t_0 = (2/3)H_0^{-1}, \tag{1}$$
   where t_0 is the age of the universe.

4. Evaluate the minimum mass (in $M_\odot$) for gravitational collapse (the Jeans mass) in the early universe at $z = 10^3$, when radiation decouples from matter. At this time, $T \simeq 3000$ K and $\rho \simeq 10^{-21}$ g cm^{-3}.
   Answer: $M_J \approx 8 \times 10^4 M_\odot$, similar to the masses of globular clusters. Hence, globular clusters may have been the first objects to form in the universe, consistent with their ages which are close to that of the universe itself.

5. Estimate the size and mass of the observable universe when the age of the universe was equal to the Planck time.
   Answer: Given the Planck time $t_p = 1.35 \times 10^{-43}$ s, the observable size was the Planck length $L_p = ct_p = 4.05 \times 10^{-33}$ cm, and the observable mass was the Planck mass $m_p = L_p c^2/G = 5.46 \times 10^{-5}$ g.

6. What are the "three pillars" of the Big Bang model?
   Answer: (1) Hubble's Law, (2) cosmic microwave background, (3) the abundances of light elements. See class notes for a description of each.

7. As the universe expanded and cooled after the Big Bang, it eventually reached a temperature $T \approx 3000$ K at which protons and free electrons combined to form hydrogen atoms. With the opacity of the gas greatly reduced due to the absence of free electrons, the blackbody emission (corresponding to $T \approx 3000$ K) was able to propagate unhindered and is currently observable from the Earth.
   (a) What was the wavelength of maximum emission when this blackbody emsission began its journey?
   (b) If we now measure it to have a peak at $\lambda_{\text{max}} \approx 1$ mm, what is the current effective temperature of the blackbody spectrum?
   (c) At what redshift must this emission have originated?
   Answers: (a) $\lambda_{\text{max}} \approx 1\mu$m, (b) $T \approx 3$ K, (c) $z \approx 10^3$.