ASTRONOMY 020

Problem Set #18

Due: March 5, 2004

1. Derive the expression for the Chandrasekhar mass,

$$M_{\rm Ch} = k_1 \left(\frac{hc}{G}\right)^{3/2} \left(\frac{Z}{A\, m_p}\right)^2 , \tag{1}$$

   where k_1 is a constant and all other terms are as defined in class. To do this, use the expression for degeneracy pressure in the relativistic limit ($v \sim c$) and the expression for central pressure in any gravitationally bound object. Keep track of all constants, unlike the simpler treatment in class, and get a numerical value for k_1. It will not be the same as the official value, 0.20, due to several approximations made in this case. Along the way, argue that if we have volume densities n_+ of ions of atomic number Z and atomic weight A, and n_e of electrons, then $\rho = Am_pn_+ + m_en_e \simeq Am_pn_+$. Also, use the fact that overall charge neutrality implies $Zn_+ = n_e$.

2. The Vela pulsar, located within the Vela supernova remnant is observed to have a period $P = 89$ ms which increases at the rate of $dP/dt = 4 \times 10^{-6}$ s/year. Assuming the pulsar has a typical neutron star mass ($M = 1.4\, M_\odot$) and radius ($R = 13.5$ km), estimate the total luminosity that you would expect to be radiated by the supernova remnant. (Hint: read discussion in Section 17-2E)

3. A black hole of mass $3M_\odot$ is accreting matter at the rate of 10^{-9} $M_\odot$/yr.
   (a) Estimate the emitted luminosity from the infalling material before it enters the event horizon, assuming full conversion of gravitational potential energy to radiation.
   (b) Alternatively, calculate the luminosity released if 10^{-9} $M_\odot$ of hydrogen gas is converted to helium every year. Show that this quantity is significantly smaller than the answer in part (a). Comment on why gravitational energy release is a more viable source of energy than nuclear fusion for an accreting black hole, but not for the Sun.
   (c) Using the answer in part (a), the effective radius of the black hole, and assuming blackbody emission, estimate the peak wavelength of observed emission. In which wavelength band does this fall?

Practice problems:

1. Zeilik & Gregory, Chapter 17, problem 4.
   Answers: (a) 9×10^{-3} m, (b) 3 km, (c) 3×10^5 km ($\approx 0.4R_\odot$) for $M = 10^5 M_\odot$, (d) 3×10^{11} km (≈ 2000 AU ≈ 0.01) pc for $M = 10^{11} M_\odot$.

2. Zeilik & Gregory, Chapter 17, problem 13.
   Answers: (a) $L = 7.1 \times 10^{25}$ W, (b) $d = 4.8 \times 10^4$ pc $= 48$ kpc. This is larger than our Galaxy, so we should be able to "see" the neutron stars in our Galaxy. However, the radiation would be peaked at x-ray wavelengths.