



A retrograde object near Jupiter's orbit

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ABSTRACT

Asteroid 2007 VW₂₆₆ is among the rare objects with a heliocentric retrograde orbit, and its semimajor axis is within a Hill sphere radius of that of Jupiter. This raised the interesting possibility that it could be in co-orbital retrograde resonance with Jupiter, a second “counter-orbital” object in addition to recently discovered 2015 BZ₅₀₉. We find instead that the object is in 13/14 retrograde mean motion resonance (also referred to as 13/-14). The object is shown to have entered its present orbit about 1700 years ago, and it will leave it in about 8000 years, both through close approach to Jupiter. Entry and exit states both avoid 1:1 retrograde resonance, but the retrograde nature is preserved. The temporary stable state is due to an elliptic orbit with high inclination keeping nodal passages far from the associated planet. We discuss the motion of this unusual object based on modeling and theory, and its observational prospects.

1. Introduction

The best-known examples of co-orbital motion in the Solar System are the Trojan clouds of Jupiter (see Barucci et al. (2002); Emery et al. (2015) for a review of their physical properties, or Milani (1993); Stacey and Connors (2008) for a review of their dynamics). Trojans are now also known for planets Earth (Connors et al., 2011), Mars (Mikkola et al., 1994; Tabachnik and Evans, 1999; Connors et al., 2005), Uranus (Alexandersen et al., 2013) and Neptune (Chiang et al., 2003; Marzari et al., 2003; Brassier et al., 2004). Other types of co-orbital motion are horse-shoe librators (first mentioned by Brown (1911)), quasi-satellites (Mikkola and Innanen, 1995, 1997), and compound orbits (first observed by Wiegert et al. (1997) and explained theoretically by Namouni (1999); Namouni et al. (1999)). In all heliocentric co-orbital motion known until recently, a small body moves under the control of the Sun and a planet in a prograde (counterclockwise viewed from above the north pole) sense, although the motion relative to the planet may appear to be retrograde.

Schubart (1978) computationally investigated certain cases of hypothetical retrograde 1:–1 mean motion resonance, in which a companion small object moves in a retrograde sense to the (assumed prograde) planet, noting that libration, an indicator of resonance, was possible.

Dobrovolskis (2012) suggested that such motion by “counter-orbitals” could be stable. Morais and Namouni (2013b), Namouni and Morais (2015), and Morais and Namouni (2016) investigated this type of motion in detail, finding a new type of stable co-orbital motion that is retrograde (Morais and Namouni, 2017). Wiegert et al. (2017) recently identified the first such retrograde co-orbital object, 2015 BZ₅₀₉, associated with Jupiter. This raises the question of whether more objects exist in retrograde co-orbital resonance. 2007 VW₂₆₆ has been known to be in retrograde motion since its discovery about ten years ago, and its semimajor axis a —less distant from that of Jupiter than the Hill sphere radius—made it possible that it was also in the 1:–1 resonance.

Recent interest in retrograde objects in the Solar System stems partly from exoplanet studies, and partly from the existence of over 80 retrograde asteroids¹ and thousands of retrograde comets. The JPL comet database² reports 1972 comets on retrograde orbits, but most (~1400) of these are Kreutz family comets, members of a split comet family that has been well-documented by the SOHO mission (Marsden, 2005). Some of our Solar System's retrograde bodies are in resonance (Morais and Namouni, 2013a; Wiegert et al., 2017) with planets.

The presence of retrograde objects within the present-day Solar System requires some explanation within current models of planet

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¹ Minor Planet Center, MPC Orbit (MPCORB) Database, <http://www.minorplanetcenter.net/iau/MPCORB.html>, retrieved Jan 12, 2017.

² <http://ssd.jpl.nasa.gov/dat/ELEMENTS.COMET>, Retrieved Mar 31, 2017.

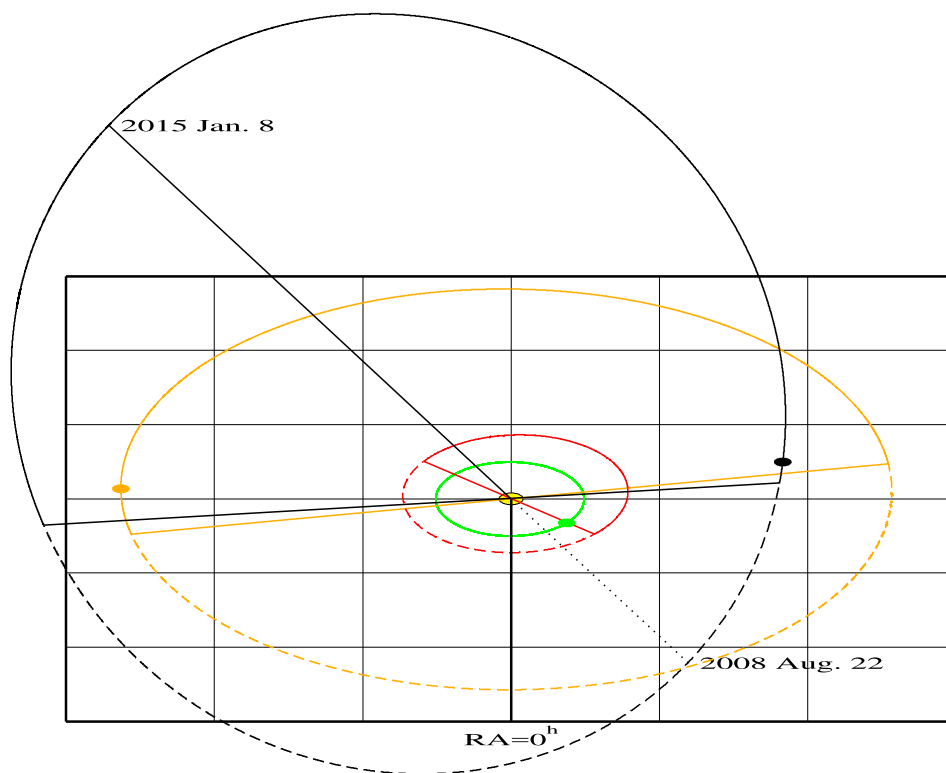


Fig. 1. Orbits of the Earth (green), Mars (red), Jupiter (orange) and asteroid 2007 VW₂₆₆ (black) over a 12.8 year period centred on the epoch of its discovery in 2007. The Sun is in the middle and a grid 2 AU square is shown in the ecliptic plane with positive X (vernal equinox) toward the bottom (axis marked by heavy line). Portions of objects’ orbits which are below the ecliptic are shown dashed, and the nodes are joined by a straight line. The positions of Earth, Jupiter, and the asteroid on the date of discovery are shown by dots on the respective orbits. The line of apsides crosses at roughly a 45° angle from lower right to upper left, shown as dotted below the ecliptic and solid above. The motion of the asteroid is clockwise due to its retrograde nature. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 1

Osculating orbital elements of asteroid 2007 VW₂₆₆ from <http://neo.jpl.nasa.gov/>, cited 23/11/2013.

Element	Name	Value	Error
	Epoch	JD 2456600.5	
a	semimajor axis	5.454 au	0.0156 au
e	eccentricity	0.3896	0.00170
i	inclination	108.358°	0.0261°
q	perihelion distance	3.32901 au	0.000586 au
ω	argument of perihelion	226.107°	0.0501°
Ω	longitude of node	276.509°	0.00114°
M	mean anomaly	146.88°	0.604°

formation, but can be understood. A near-Earth asteroid population with retrograde objects was produced from main belt sources in models by Greenstreet et al. (2012), with the majority originating from 3:1 resonance. These authors stressed that an integrator having good numerical characteristics for close encounters was essential for matching even the small number of observed objects. On the other hand, the outer solar system Damocloids, which include a significant proportion of retrograde members, were proposed by Jewitt (2005) to originate from long-period comets, on both dynamical and compositional grounds. The long-period comets, coming from low angular momentum states in the Oort cloud, contain a high proportion of retrograde orbits due to interactions with passing stars and the Galactic tidal field e.g. Wiegert and Tremaine (1999); Morais and Namouni (2017).

The dynamics of nearly-coplanar ($i \approx 163^\circ$) retrograde co-orbital 2015 BZ₅₀₉ are essentially understood (Morais and Namouni, 2016) and show surprising stability. We find that 2007 VW₂₆₆ — retrograde but far from coplanar ($i \approx 108^\circ$) — does not share that stability, but displays a new form of retrograde temporarily protected orbit.

2. Configuration of the present orbit

Asteroid 2007 VW₂₆₆ was discovered³ on Nov. 12, 2007 (UT) by the Mt. Lemmon Survey, at a magnitude $m \sim 21.4$, with 60 subsequent observations spanning 38 days.⁴ The result is a nominal orbit shown in Fig. 1. It is shown below that the behaviour over about 10,000 years is well described, but due to Jupiter encounters, not well known outside of that range. The osculating elements for epoch 2456600.5 (2013-Nov.-04.0) TDB, with standard errors, are summarized in Table 1. Since there have been no observations since that date, the only orbital change is due to interactions that are included in our models, and we have based our calculations on these initial conditions. The data arc is sufficient to determine the orbit with a well defined error model (Milani and Gronchi, 2010), allowing statistical investigations such as the clone orbits described below.

The eccentricity e of roughly 0.39 means that the orbit is elongated, while at roughly 108° , the inclination is large and motion is retrograde. Mean motion resonant interaction with Jupiter may be observed for cases where a is within roughly a Hill radius, or about 0.355 AU, of its semi-major axis $a_J \sim 5.2$ AU, and this criterion is met at about the one σ level. Due to the retrograde orbit, this may be regarded as suggestive of possible strong interaction, but not a mean motion resonance in the usual sense. It is thus useful to investigate the stability of the orbit and whether it has “counter-orbital” (Dobrovolskis, 2012; Morais and Namouni, 2016) stability. The large absolute value of the inclination $|i|$ and of the eccentricity suggest that the Kozai (1962) mechanism might operate. Use of the Kozai formulae (Connors, 2014) with $|i|$ suggests that it should, however numerical integration did not bear this out. This aspect is not

³ <http://www.minorplanetcenter.net/mpec/K07/K07W21.html>, cited 1/12/2013.

⁴ <http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2007+VW266&orb=1>, cited 1/12/2013.

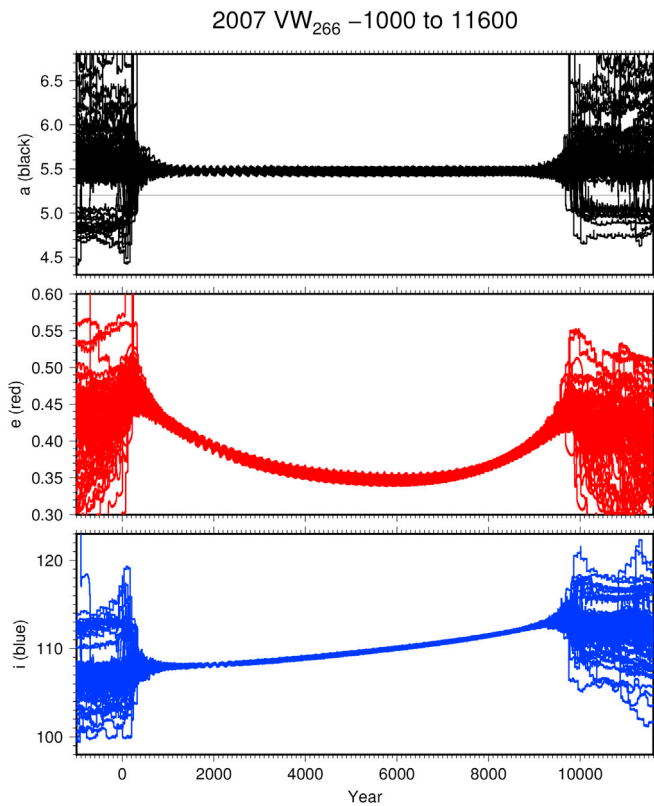


Fig. 2. Evolution of the semimajor axis a (top:black), eccentricity e (middle:red), and inclination i (bottom:blue) in the time range 1000 BCE to 11600 CE (years are based on division of JD differences by 365.26), for 101 Gaussian distributed clones of asteroid 2007 VW₂₆₆. The closeness of the orbits shows stability for approximately 10,000 years. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

further investigated.

The present counter-orbital behaviour appears to allow avoidance of close encounters with Jupiter, which along with a being within a Hill sphere radius of that of Jupiter, could be an indication of 1:1 retrograde resonance. We find that instead 13/14 retrograde mean motion resonance with Jupiter (13/-14) was operative, and discuss this below. Other aspects of the present behaviour were best investigated by a clone study.

3. Statistical study of current counter-orbital behaviour

The elements shown in Table 1 have a relative accuracy of order 10^{-3} . The extent to which such elements can be used to describe the real object could be questioned. A study of “clone orbits” derived using elements generated using the uncertainties obtained in reduction of the nominal orbit can establish whether it indeed is reliable. 101 clones were generated as Gaussian deviates (Press et al., 1992) with the standard deviations indicated, and integrated using *Mercury* (Chambers, 1999) with the embedded RADAU integrator, with one day time-step and 26 day output interval, from the present backward and forward roughly 30,000 years. All planets and Pluto were included. Independently, a set of 101 different clones were generated using the covariance matrix and integrated, as described by Wiegert et al. (2017), to give statistically indistinguishable results. The results for semimajor axis a , eccentricity e , and inclination i for all 101 clones are shown in Fig. 2. In the years 0–10,000, the clones are tightly bunched together. This indicates that the behaviour is well determined by the present nominal orbit during this period. It is bounded by rapid changes in the parameters of all clones, caused by close approach to Jupiter. With the present integrator and orbital uncertainties, the behaviour past the encounters may be discussed

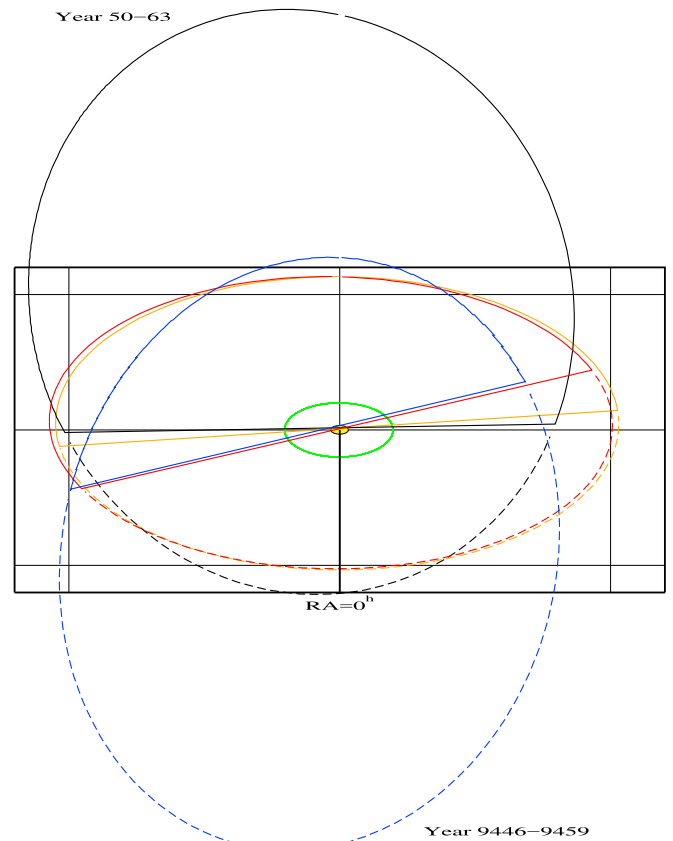


Fig. 3. Three-dimensional view of the orbit illustrating perihelion advance. The orbit is viewed from 30° above the ecliptic plane. The orange and black lines are the orbits of Jupiter and 2007 VW₂₆₆, respectively, for the years 50–63, and may be compared to the orbits shown in Fig. 1. The red and blue orbits are for Jupiter and the asteroid, respectively, in 9446–9459. The nodes are shown as in Fig. 1, and have not advanced significantly for Jupiter nor the asteroid. Grid lines shown in the plane of the ecliptic 5 au from the Sun, while the outline box extends to 6 au. Earth orbit (green) for scale is schematic. For clarity, the (greatly changed) apsidal lines are not shown but it is clear where they are. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

in only a statistical manner. The present orbit changes slowly, maintaining $a \sim 5.5$ au during the entire stable period, with e having been about 0.45 at the times of entry and exit from the stable period, decreasing to roughly 0.35 in about 6000 CE, and rising again after that. The inclination was retrograde throughout, initially near the present value of 108° , and increasing to about 113° by the time of exit from the stable period.

All clone orbits entered and left the “stable state” through a close encounter with Jupiter. Fig. 3 shows the orbit, and that of Jupiter at two times, near the time of injection to, and exit from, the stable state. The descending node was near Jupiter’s orbit at the time of injection. Mainly due to rotation of the line of apsides, the ascending node will be near it when the object is ejected. The previous states are not entirely random, however. This is most noticeable for a shown at the top of Fig. 2. For brief periods at the times of scattering, i.e. ca. 100 CE and ca. 9800 CE, a has a range of values centred on 5.5 au: before and after these times, values near 5.2 au, the semimajor axis of Jupiter, are avoided, although there can be brief traversals of this value. Longer-term results for the nominal orbit (not shown) have a general tendency to avoidance, and stability over about 60,000 years. Until the orbit is better determined, this intriguing aspect cannot be meaningfully investigated further for this object. Since all clones show essentially the same behaviour, further, theory-based discussion of the nominal orbit is useful. Our main aim is to explain the rate of rotation of the apsides since this determines the

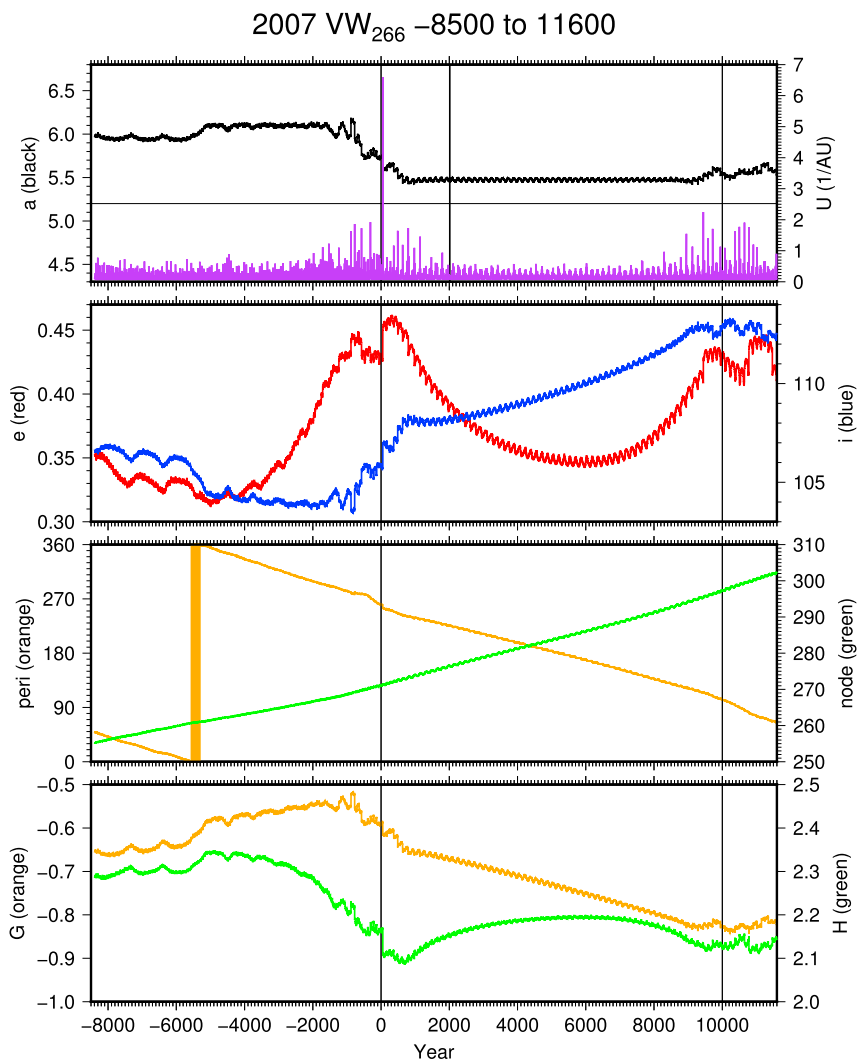


Fig. 4. Parameters of the nominal orbit. The semimajor axis is shown in the top panel (black) with the inverse distance to Jupiter (purple) in units of au^{-1} . The middle panel shows eccentricity (red) and inclination (blue), while the second bottom panel shows the argument of perihelion (orange) and the longitude of the node (green). The bottom panel shows their respective Delaunay parameters $G = \sqrt{a(1 - e^2)}$ (orange) and $H = G \cos i$ (green). The black vertical lines indicate the rough limits of resonant capture, while a single vertical line in the top panel marks the present era. Note the use of separate scales on right and left for all quantities. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 2
Changes in Keplerian and Delaunay elements of asteroid 2007 VW₂₆₆ during the stable period.

Element	1000 CE	9000 CE	Rate
<i>a</i>	5.46	5.46	0 au/yr
<i>e</i>	0.430	0.398	nonlinear
<i>i</i>	108.1	112.6	0.0005625°/yr
ω	239	122	-0.0146°/yr
Ω	273.8	294.0	0.00253°/yr
<i>G</i>	-0.66	-0.82	2×10^{-4}
<i>H</i>	2.11	2.14	nonlinear

duration of the “stable state”.

4. Behaviour of the nominal orbit

The nominal orbit whose parameters are shown in Fig. 4 follows that of the clones in the years ca. 0–10000 CE (i.e., the “stable state”) in which the latter coherently indicated the behaviour. Outside of this period, the behaviour is merely indicative, and suggests a near-resonant Centaur orbit. Retrograde Centaurs have been discussed by Morais and Namouni

(2013a). While in the stable state, *a* varied little, with only repetitive, small amplitude, short-period (about 75 year) variations. The sidereal period while in this state is 12.74 years, while that of Jupiter is 11.86 years. The period difference causes the relative configuration to repeat after about 170 years. However, a passage close to Jupiter can occur near both nodes, so that the short-period oscillation is about half this. These passages are seen in the top panel of Fig. 4 as spikes in the inverse distance to Jupiter with about the same period as that of the small variations in *a*. As shown in Table 2, based on graphical measurement, the inclination *i* changed from about 109° to 112° during the stable period, i.e. not very much, while the eccentricity *e* was about 0.45 at entry, 0.35 near the middle, and 0.43 near its end. The node was near 270° during the entire period studied, however the argument of perihelion was regressing rapidly, with a value near discovery of about 230°. This rapid regression dominated the motion, since the close encounters shown in Fig. 3 arose through geometry change mostly associated with it.

Based on its *a* of 5.454 au, we suspected that 2007 VW₂₆₆ could be in the 13/14 retrograde mean-motion resonance, (13/-14 in the notation of Morais and Namouni (2013a)). The critical argument ϕ for the *p*/*q* retrograde resonance is, from Morais and Namouni (2013a),

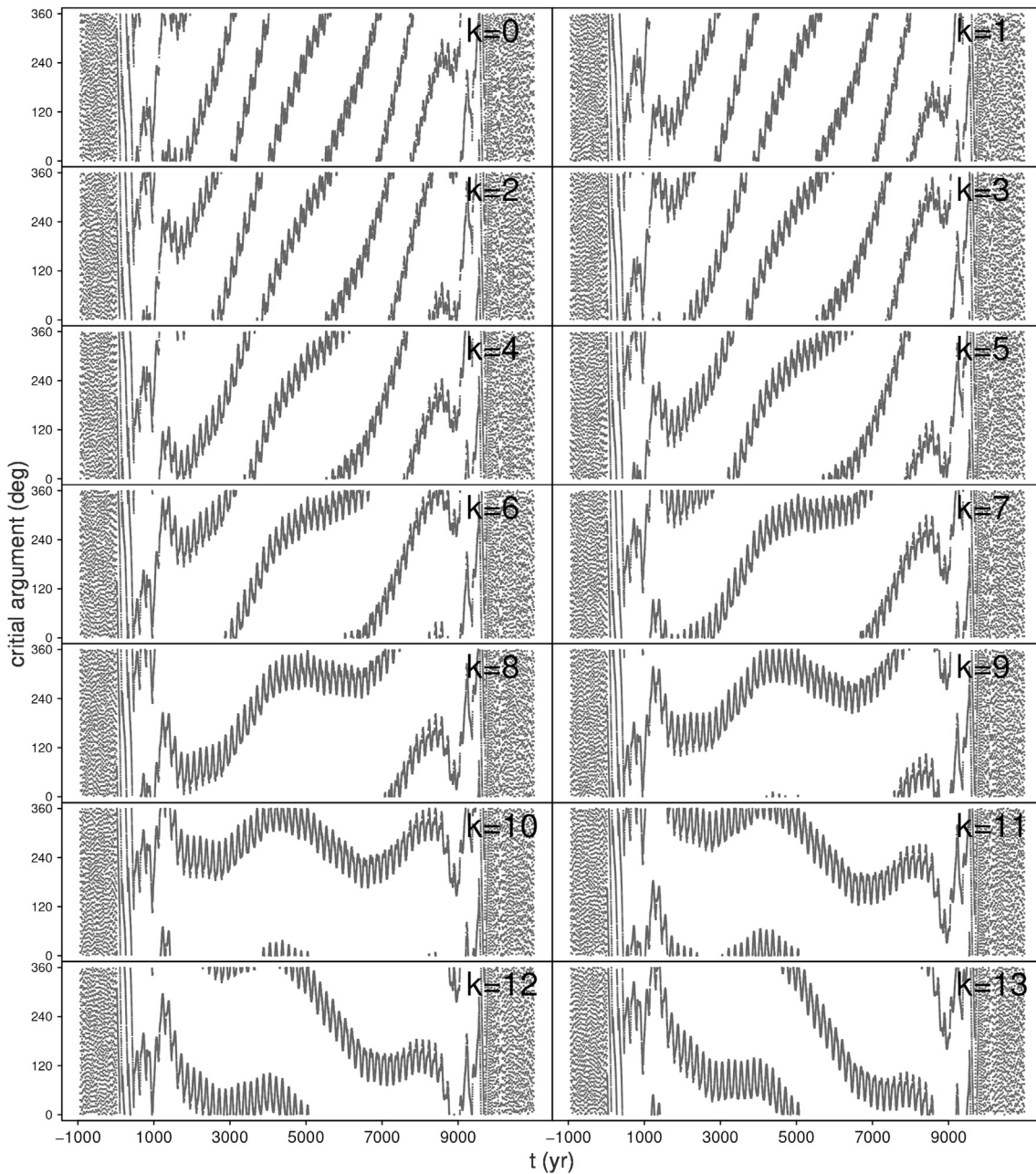


Fig. 5. Critical argument of the 13/-14 mean motion resonance for asteroid 2007 VW₂₆₆ for different values of k .

$$q\lambda^* - p\lambda' - (p + q - 2k)\varpi^* + 2k\Omega \quad (1)$$

for $k = 0, 1, 2, \dots, (p + q)/2$. The $k = 0$ term is the strongest in the planar retrograde case, as the terms are proportional to $\sin(i/2)^{2k}$ where i is the inclination. In our case, $\sin(i/2) \approx 0.59$ and so terms with $k > 0$ may not be negligible. In fact, when we plot ϕ for different values of k we obtain Fig. 5, where the $k = 0$ critical argument is found to circulate slowly, while larger values do so even more slowly or librate (e.g. $k = 10$) during the course of the resonance capture. We also note that the $k = 13$ case is numerically equal to the prograde planar critical argument, and that it circulates more slowly than the retrograde one. We conclude that the 13/-14 resonance is active during this period. We did not find any evidence for sustained libration in the critical arguments of the nearby 12/-13 or 14/-15 resonances. The 13/-14 resonance is nominally at $a \sim 5.468$ au, which is only 0.264 au from that of Jupiter ($a_J \sim 5.204$ au). The slow change of this resonant argument, as seen in Fig. 5, suggests that 13/-14

mean motion resonance dominates the motion, so that a theory of motion in that resonance may be used.

We thus examined the expected behaviour analytically with an expansions of the disturbing function (e.g. Murray and Dermott (1999) Ch 8). Our expansion to second order in the inclinations and eccentricities gave a precession period for $\dot{\omega}$ of 16,000 years, in rough agreement with the 25,000 years that arises from modeling. Namouni and Morais (2017b) discuss why such standard expansions of the disturbing function cannot work well for high inclination and eccentricity, and provide new expansions. Namouni and Morais (2017a) suggest that such expansions can be manipulated with symbolic algebra software to provide usable series. These two articles further provide several examples of high inclination motion. A more intuitive approach is provided by (Moulton, 1914), notably in Chap. IX. However, this approach does not lead to numerical values. The rough agreement we have obtained, and this near-concurrent development of relevant techniques, suggest that analysis (beyond the scope of the current article) could address the essential

aspect of the current motion of 2007 VW₂₆₆, which is that rapid regression of the argument of perihelion has placed 2007 VW₂₆₆ into a state where it cannot scatter from Jupiter, and will eventually place it into a state in which this is inevitable.

The protection mechanism discussed here is necessarily temporary. If the rate of regression is not high, other scattering is likely immediately after the initial one. Since the rate of regression is high, the geometry again becomes one favoring scattering, after a relatively short time. Long-term stable avoidance (Peale, 1976) can be favored by resonance, as in the case of Pluto (Cohen and Hubbard, 1965). Here the effect is more dramatic but short-lived. In principle it could also work in the prograde case. We have found no previous discussion of this, nor have we yet investigated potential examples.

Our aim in this paper of understanding the motion of 2007 VW₂₆₆ has led us to find a second retrograde protection mechanism, beyond the actual retrograde 1/-1 resonance (Morais and Namouni, 2016; Wiegert et al., 2017), albeit capable of acting only temporarily. It is another example of retrograde near-co-orbital and capture orbits meriting more study (e.g. Morais and Namouni, 2017; Namouni and Morais, 2017b, a).

5. Observational considerations

Asteroid 2007 VW₂₆₆ was observed for 38 days in 2007 when near perihelion, opposition, and the ecliptic, but has not been recovered since. Although it was observed for a relatively short period of time, we showed above that its orbit is well enough known to allow definitive study for about 10,000 years. Since its approaches to Jupiter at injection and expulsion from the protected orbit are not necessarily very close, it is possible that the known period of the orbit could be extended through observational recovery, which would reduce the statistical uncertainty compared to that used in our studies. We proceed to briefly discuss recovery prospects.

The object was discovered close to the ecliptic and at a northerly declination, and moved southward remaining relatively bright. Despite favourable viewing circumstances, it was not observed after Dec. 20, 2007. It proceeded to perihelion on Aug. 22, 2008, shown on Fig. 1, by which time it was at southerly declination. It remained poorly placed for northern hemisphere observers, crossing again into northerly declinations only in May 2013. At aphelion on Jan. 8, 2015, it had $m \sim 24.4$, which is about as faint as it can be. There is a trade-off in observational circumstances at this time. Although the object becomes brighter as it moves on average closer to Earth, the buildup with time of positional error in the orbit, and geometric nearness, mean that the 3- σ errors in on-sky position become larger. In Jan. 2014, the apparent magnitude was about 24.3, and the 3- σ errors in Dec. and RA roughly $0.2^\circ \times 0.75^\circ$. At aphelion these errors were slightly larger at $0.3^\circ \times 1.0^\circ$, and a year later in 2016, $0.3^\circ \times 1.5^\circ$. In Jan. 2020, with $m \sim 21.5$, recovery by surveys can be expected, but the search box will have grown to about $5^\circ \times 6^\circ$, making a dedicated search difficult. It is thus suggested that recovery in the present epoch with a small amount of dedicated search time would be worthwhile.

6. Conclusions

Asteroid 2007 VW₂₆₆ has a highly inclined, retrograde orbit, which is in a 13/-14 mean motion resonance with Jupiter. A temporary protection mechanism is acting to prevent its scattering by Jupiter in the present epoch. However, the very motion of the argument of perihelion that has led to the elliptical orbit having a geometry not bringing the object near Jupiter will eventually place it into a configuration where this does happen. The stability period of order 10,000 years is very short on the timescale of the solar system, so we may not expect large numbers of objects to share this type of orbit. In contrast, the motion of the true counter-orbital object 2015 BZ₅₀₉ (Wiegert et al., 2017) is nominally stable, so there may be larger numbers of such objects awaiting discovery. While surveys will undoubtedly find more objects in retrograde

resonant orbits, and eventually recover 2007 VW₂₆₆, this intriguing object could be studied in more detail now with a search that is likely quite feasible.

The properties of the orbit presented here may be applicable to other high inclination, eccentric objects (retrograde or prograde). Longer-term orbit analysis may be possible upon observational recovery, since approaches to Jupiter are not extremely close. The present discussion basically explains the temporary counter-orbital stable state, but the precise origin of 2007 VW₂₆₆ cannot be determined based on this and the observations currently available. Recent advances in theory for high inclination, high eccentricity objects will prepare us for more of them being found with improved modern surveys.

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Appendix A. Supplementary data

Supplementary data related to this article can be found at <https://doi.org/10.1016/j.pss.2017.11.009>.

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