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The Discovery of Earth's Trojan Asteroid

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Abstract

Trojan asteroids share the orbit of an associated planet by moving slowly about one of the two Lagrangian points that are located 60° ahead of or behind the planet, along its orbit, as measured from the Sun. This orbital configuration was proposed in 1772 by Lagrange as a solution for the motion of three bodies. The first Trojan asteroid was discovered, associated with Jupiter, in 1906. Many thousands of Jovian Trojans are now known to exist, several for Mars and Neptune, and one for Uranus. With calculations showing that Earth could have stable associated Trojans, but with observations being very difficult, it is only in 2010 that the first Earth Trojan was discovered. That body, 2010 TK_7, has an extreme form of Trojan orbit that allows it to move far from the L_4 Lagrangian point with which it is associated, and sometimes even to jump to the other Lagrangian point. The circumstances of discovery, dynamical behaviour, and context are discussed.

Résumé

Les astéroïdes troyens partagent l'orbite d'une planète associée en se déplaçant lentement autour de l'un des points triangulaires de Lagrange, qui sont situés sur son orbite, à 60° en avance ou en retard de la planète, mesurés par rapport au Soleil. Cette configuration orbitale comme solution pour le mouvement de trois corps fut proposée en 1772 par Lagrange. Le premier astéroïde troyen a été découvert, associé à Jupiter, en 1906. Plusieurs milliers de troyens de Jupiter sont maintenant connus, plusieurs de Mars et Neptune, et un seul d'Uranus vient d'être trouvé. Des calculs montrent que la Terre pourrait posséder des astéroïdes troyens stables, mais les observations sont très difficiles, donc ce n'est qu'en 2010 que le premier astéroïde troyen de la Terre fut découvert. Ce corps, 2010 TK_7, possède une forme extrême de l'orbite qui le permet de se déplacer loin du point L_4 de Lagrange avec lequel il est associé, et même de parfois passer à l'autre point de Lagrange. Les circonstances de découverte, le comportement dynamique et le contexte sont discutés.

Introduction: Workings of the Solar System

The name "Trojan asteroid" seems to interest people by evoking an association with history, myth, and heroism. Yet it is an enigmatic term: in what way can an asteroid, a small celestial body, be "Trojan"? To understand Trojan asteroids, and in particular the recently discovered "Earth Trojan" provisionally named 2010 TK_7, it is useful to delve back into the history of astronomy. This allows us to see how our approach to understanding the dynamics of our Solar System has evolved over time and the recent overlap of astronomy with chaos theory, as manifested intriguingly by our new orbital partner.

Although this brief history of Solar System astronomy is a personal view, fact-checking has been done with Petersen (1993), which is a recommended source, blending history with chaos theory to illustrate modern ways of understanding celestial mechanics. Moulton (1914) also has useful historic notes on the development of the subject to his time, interspersed with the textbook's material on mathematical astronomy.

The early history of astronomy was largely concerned with measurement and timing. The ability to predict seasonal, monthly, and daily cycles had an essential practical role in agrarian societies. Some ancient societies, such as the Babylonians and Chinese, catalogued and predicted planetary motions for astrological purposes. Compared to unpredictable systems like the weather and earth movements, the sky could be understood, which may have given comfort in chaotic lives subject to the vagaries of nature.

There was every reason in ancient times to suppose that all of the motions in the heavens, observed from Earth, were in fact centred on Earth (geocentric). For Earthly beings observing objects held to Earth's surface by gravity and slowed by friction, the local region seems to be a good frame of reference. Geocentric motion was implicit in most ancient theories, most famously codified by Claudius Ptolemy of Alexandria about AD 100. The Ptolemaic system allowed quite accurate calculations, albeit for a limited duration. It formed a mechanistic system that served well for nearly 1400 years.

The "paradigm shift" of Copernicus in stating that the Universe was centred on the Sun rather than on or near the Earth was partly motivated by esthetics (Gribbin, 2002). Copernicus was aware of both ancient and nearly contemporary heliocentric systems (Kuhn, 1957). He proceeded to claim that Earth—long known to be a sphere—by rotating, could explain the daily motion of the heavens. From there to motion of the Earth in space was not a large step. However, Copernicus was also motivated by the desire to make an easier and more accurate computing system, and to resolve some inconsistencies, such as the lack of observed apparent change in size of the Moon, that arose in the Ptolemaic system. His replacement Sun-centred system retained mechanical features, such as circular motions, from the old system. It did not form part of a dynamical system in which forces needed to act. Our modern understanding is based on forces, mostly the force of gravity.

Perhaps motivated by astrological beliefs (Petersen, 1993), Kepler sought out some driving force for planetary motions in order to get away from the "circles within circles" that characterized both the Ptolemaic and Copernican systems. His parallel quest for more accuracy in the Copernican system was partly based on the high quality of the observational material left to him by Tycho Brahe. His initial breakthrough was to realize that planetary orbits around the Sun could be described by ellipses in a plane with the Sun at one of the foci. This is Kepler's First Law, and the parameterization of orbits by ellipses (whose characteristics may change slowly in time) is used in describing orbits today. Figure 1 shows the present elliptical orbit of the Earth Trojan asteroid 2010 TK_7 around the Sun. The special nature of this orbit is subtle, and in many ways it has an orbit typical of other asteroids, only nearer to the Sun. Kepler's Second Law—that within its orbit, a body moves such as to sweep out equal area per unit of time—is also nicely illustrated by this somewhat elliptic orbit. Kepler's Third Law states that for different orbits, the period of revolution about the Sun is proportional to the 3/2 power of the mean distance from the Sun (often stated in the form $P^2=a^3$). The first two laws were in Kepler's *Astronomia Nova*, published in 1609, and the Third Law was expounded in *Harmonice Mundi* in 1619. Despite the likely motivation of seeking a force-based or dynamic system, Kepler in the end gave an essentially modern description that did not contain dynamics.

From the point of view of the motion of bodies in the Solar System under the influence of forces, Newton accomplished what had eluded Kepler. Not only was he a co-inventor of calculus, which gave greatly enhanced mathematical tools, but he formulated the law of gravitational force. Kepler's laws could be derived from it and his laws of motion (Newton's three laws). Much of this work was presented in the *Principia*, published in 1686, but already in 1684 he had shown that the inverse-square law of gravity led to Kepler's elliptical orbits (Petersen, 1993). Newton's solution of the two-body problem, such as the motion of one planet around the Sun, is exact. It remains remarkable that, with the limited exception of the Lagrange three-body solution detailed below, it is the *only* exactly solvable problem in classical gravitational mechanics. Calculus, however, provided a mechanism for approximation, allowing numerical solutions of arbitrarily great precision to general problems of celestial mechanics. Among these may be mentioned the theory of the Moon, whose orbit around the Earth is greatly perturbed by the gravitational influence of the Sun, and that of the planet Uranus, accidentally discovered in 1781 by William Herschel.

Observations, greatly enhanced by telescopes since their invention around 1600, allowed the recognition that the orbit of Uranus was affected, or perturbed, by some unknown body. The growing body of observations, coupled with advances in computational mathematics, led to the prediction of the existence of a perturbing body, and the discovery of the planet Neptune in 1846. The predictions may have been erroneous and the discovery in some ways fortuitous (Petersen, 1993),

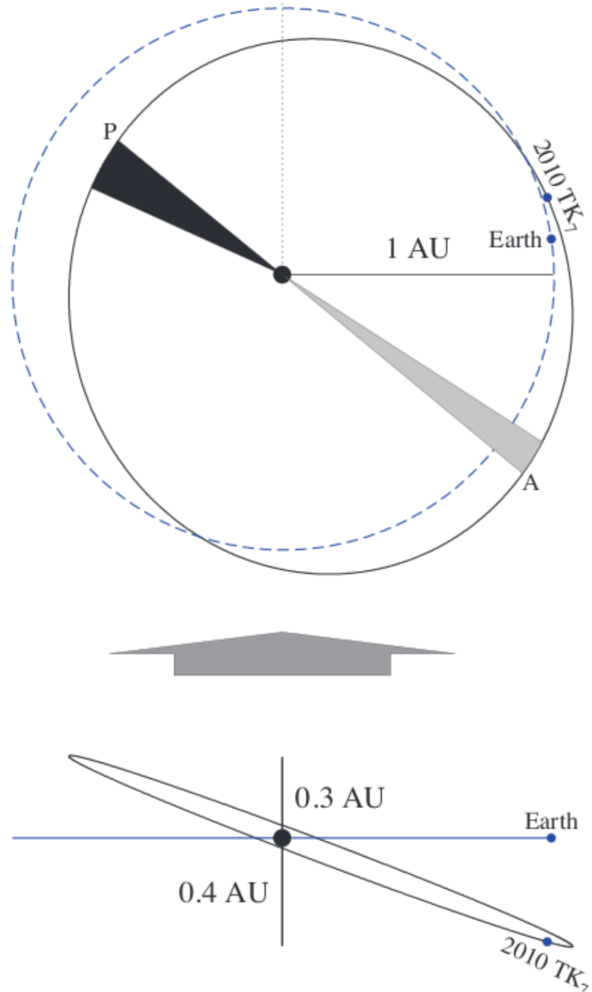

Figure 1 — Views of the orbits of Earth (dashed) and asteroid 2010 TK_7 as seen from above the north ecliptic pole (top) and looking from the side in the direction of the large grey arrow. The X axis is along the solid line 1 AU long on the right. The Y axis follows the dotted line upward. Earth and the asteroid are indicated by dots at their positions at the time of discovery of 2010 TK_7, 2010 October 1. The central dot represents the Sun and the dots are not to scale. Two of Kepler's laws are illustrated in this figure. The orbits are ***ellipses*** *in their own plane. Earth's orbit is very nearly circular. The orbit of the asteroid is more clearly elliptical although slightly distorted when seen from above, and very distorted when seen from the side. In the top view, the aphelion of the asteroid is marked A, and the perihelion P. Ten days of motion are shown near each point. The areas swept out in these equal periods of time, shown by shading (black near perihelion, gray near aphelion) are equal by Kepler's Second Law, the* ***law of areas****. The speed of a body must thus be larger near perihelion. In the bottom panel, the inclination of 2010 TK_7 with respect to Earth's orbit is clear, allowing excursions of roughly 0.3 and 0.4 AU above and below the plane.*

but the discovery of Neptune was still a triumph of celestial mechanics, and the years near 1800 stand as a golden age of mechanistic astronomy. Green (1999; p. 341) cites the great French mathematician and astronomer Laplace as stating that if all the forces and data were available for analysis "nothing would be uncertain, and the future, like the past" could be precisely predicted. Quantum mechanics imposed a vastly different, probabilistic worldview, starting about 1900, with most differences being on the microscopic scale. However, even in celestial mechanics, the lack of exact knowledge of initial conditions, and accumulation of small errors, make Laplace's mechanistic view of planetary dynamics break down due to chaotic effects (Lecar *et al.*, 2001; Wisdom, 1987).

An elegant insight of mechanistic astronomy, relevant to the Trojan asteroid problem, derived from the study of the motion of the Moon. Joseph Louis Lagrange (1736-1813) was born in Turin, Italy, into a French family. His later career as a mathematician and *géomètre* (practitioner of celestial mechanics) unfolded in Berlin and Paris. He responded to a call by the Paris Academy in 1772 for a contest "perfecting the methods on which the lunar theory is founded" (Wilson, 1995). The complex motion of the Moon under the influence of both Earth and Sun (*i.e.* a three-body problem) had great practical interest, with applications in navigation and surveying.

Lagrange's more general approach to the theory of three bodies led to a prize-winning entry entitled *Essai sur le problème des trois corps* (Lagrange, 1772). Lagrange's solution did not require the third body to be of negligible mass, and gave two classes of solution involving special points now known as *Lagrangian Points*. The geometry is shown in Figure 2. In one class, the three bodies could be along the same line (collinear). What came to be called the L_1 and L_2 points are located near the planet, and these are of modern technological use for satellites. The third collinear point, L_3, is found on the opposite side of the Sun from the planet. The collinear points are not stable: spacecraft need to actively control their position to stay near them. Earth Trojan asteroid 2010 TK_7 is the first body known to be able to temporarily reside at L_3 (Connors *et al.*, 2011). The other class of solution has the smaller bodies at positions forming an equilateral triangle with the planet and Sun; in other words, as viewed from the Sun, these positions are 60° ahead of or behind the planet in its orbit. The point leading the planet in its counterclockwise (viewed from the north) path around the Sun is called L_4, and that following the planet is called L_5. These triangular points allow stable motion of a small body under the influence of the Sun and planet. Since they are very near the planet's orbit, and the small third body stays near them, the orbit is "shared" at least on average, and the motion is referred to as "co-orbital."

As envisaged by Lagrange, these solutions were elegant: an abstract, mechanical picture of perfection. Comets were known, but their known orbits at the time[1] would not have led

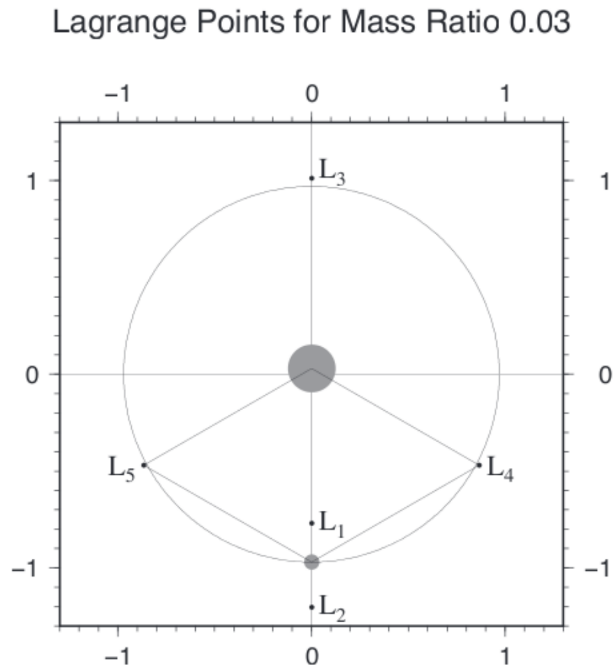

Figure 2 — Geometry of Lagrangian points. The five Lagrangian points are located in space relative to the planet (lower black dot), as it orbits a star only 33 times more massive (larger dot near origin), as seen in a frame revolving with the planet. In this co-rotating frame, the planet is considered stationary so that motion may be shown with respect to it. The inner Lagrangian point L_1 is inward of the planet, the outer one L_2 further from the star. The opposite Lagrangian point L_3 is on the other side of the star. The L_4 and L_5 triangular points are the corners of equilateral triangles with the star and planet, and are seen as 60° from the star if viewing from the planet. Trojan asteroids can remain near these points, but generally are not exactly at them. The planet orbits the origin (centre of mass) on a circle of scaled radius 1, and the star also orbits this point, but on a small circle that is not shown.

one to expect them to be found at Lagrange points. Asteroids were discovered only in 1801, 29 years after Lagrange's publication. Thus, Lagrange regarded his special solutions as purely of theoretical interest ("*Cette recherche n'est à la vérité que de pure curiosité*") and not existing in the real Universe ("*ces cas n'aient pas lieu dans le Système du monde*"). He was not to be disproved until 1906, when the first Trojan asteroid was discovered, associated with the planet Jupiter.

Trojan Asteroids

The first minor planet was discovered on 1801 January 1, by Giuseppe Piazzi in Palermo, Italy (Cunningham *et al.*, 2011). It was subsequently numbered and named 1 Ceres. It is now considered to be a dwarf planet, but retains its asteroid or minor-planet number. The nature of Ceres was subject to debate after its discovery, and it was initially referred to as a "planet" (Cunningham *et al.*, 2011). The term "Planet" was still found in the German asteroid literature over a century later. After William Herschel found the second asteroid Pallas in

1802, the term “asteroid” came into use for these objects, which were of star-like appearance even in his telescope, large for the era (Cunningham *et al.,* 2009; Cunningham and Orchiston, 2011). As the 19th century advanced, it was found that large numbers of such bodies existed, mainly in a zone between the orbits of Mars and Jupiter.

The Königstuhl Observatory on the small mountain of the same name overlooking Heidelberg was among the world’s foremost in asteroid discovery in the late 19th and early 20th centuries. Astronomer Max Wolf (1863-1932) developed advanced photographic and search techniques using its large refractors and what was for a short while one of the world’s largest telescopes, the 71-cm reflector. This observatory had discovered 316 asteroids by 1914, and in 1913, it found 32 of the year’s 88 new asteroids (Hills, 1914). Wolf’s work spanned many areas of interest in the early 20th century, but a common theme was the efficient application of modern imaging techniques, at that time, photography and plate scanning (Kopff, 1928). Some of Wolf’s early methods are described in English in considerable detail by Holden (1896): by the early 20th century, these techniques had been yet further advanced. Close cooperation with Johann Palisa of the Vienna Observatory allowed rapid visual followup of asteroids discovered photographically and with optomechanical aid by Wolf’s group at Heidelberg (Freiesleben, 1962).

In the course of routine observations on 1906 February 22, Wolf and August Kopff discovered four asteroids (Wolf, 1906a). Wolf himself discovered what was denoted in the system of the time, 1906 TG, drawing attention to its small movement in right ascension. Kepler’s Third Law gives a longer period, and thus smaller motion, for an object near the distance of Jupiter than for the closer main-belt asteroids commonly discovered. At the time of discovery, 1906 TG was nearly in opposition: the anti-sunward region of the sky is optimal for asteroid searches since objects there show near-full phase and are about as near as possible to Earth, making them brighter. In addition, Wolf’s method relied on initially looking for streaks on photographic plates, as at this point, the apparent motion of asteroids is rapid, although retrograde, making a longer streak than at other positions along the orbit. The object was near, but not exactly at, the Jupiter L_4 Lagrangian point, preceding Jupiter by about 70° in their nearly common orbit. This was not apparently realized at the time of discovery, when most asteroids being found were main-belt objects. The second Trojan was found by Kopff among seven new discoveries (Wolf, 1906b) on 1906 October 17, designated initially 1906 VY. It was near opposition and about 40° from Jupiter near the L_5 trailing Lagrangian point. On 1907 February 10, Wolf (1907) discovered 1907 XM, once again near opposition, but about 120° from Jupiter near the L_4 point. The Heidelberg discoverers (Wolf & Kopff, 1907) grouped these unusual asteroids, following a suggestion by Palisa to assign them names from Homer’s *Iliad*, the epic poem about the Trojan War. Thus arose the term “Trojan” asteroid. Perhaps cautiously, or perhaps not feeling that these bodies corresponded to the Lagrange special solution, the Heidelberg group did not mention the likely connection to Jupiter, noting them as merely “*sonnenfernen*” or far from the Sun. By this time, the orbit of 1906 TG had been well enough determined to assign its current number and name as 588 Achilles. 1906 VY was named after Achilles’ dear friend or cousin Patroclus and later assigned the number 617, and 1907 XM was named for their mutual enemy, the Trojan Hector. The latter was later numbered as 624, and its name is now spelled Hektor. Subsequently, asteroids near the Jupiter L_4 point have been named after Greek heroes, and those near L_5 after Trojans. The early naming did not follow this convention, and Trojan hero Hektor is near L_4, while Greek hero Patroclus is near L_5. Generically, the Jupiter Lagrange-triangular-point asteroids are now called Trojans. Now that asteroids are known near the Lagrangian points of other bodies, the term is extended by giving the name of the guiding body, as in “Earth Trojan.”

It is unclear to what degree the Heidelberg observers were aware of the three-body solution of Lagrange. Certainly, they were aware of important methods in perturbation theory of asteroids that are an important part of Lagrange’s overall works on celestial mechanics. However, one person was very well placed to interpret the new discoveries: the Scandinavian astronomer Carl Ludwig Charlier. (K.L., 1935). He had recently published on the topic of treatment of the three-body problem with planetary perturbations (citation in Charlier (1906), not available to the present authors) and had just written a two-volume book on celestial mechanics (Charlier, 1902, 1907) that introduced the currently used designations of the triangular Lagrangian points. Already in May 1906, Charlier (1906) noted the great interest of 1906 TG, in that its rate of motion, commented on as slow by the discoverers, was very close to that of Jupiter. He pointed out that it was close to, but not exactly at, a triangular point, but that eccentricity and inclination of orbits would lead one to expect slight differences. He added a description of the epicyclic and librational motions that could be expected to be observed over the long term (the libration has a period of 148 years). These aspects are discussed in more detail below. Finally, he pointed out that 1906 TG could belong to a new class of bodies, and that it would be a good idea to search the other Lagrangian point. Despite the journal of publication, *Astronomische Nachrichten*, being heavily used by astronomers in Heidelberg, it is unclear what degree of credence was given there to Charlier’s rapid and correct interpretation of the situation.

There are now over 5000 Trojan asteroids with relatively well-established orbits. The Lagrange theory is widely known and certain aspects of it are used in space navigation. Trojans are known for all the planets except Mercury, Venus, and Saturn, and even for some asteroids/dwarf planets. We now pass to the circumstances leading to discovery of Earth's Trojan companion.

Mars Trojans and Assorted Co-orbital Objects

The recognition in 1990 of 5261 Eureka as the first Trojan not associated with Jupiter was described by Innanen (1991) in this *Journal*. Eureka is a fitting name to be associated with something new, and this Mars Trojan seemed to stimulate interest in finding other co-orbital bodies. Similarly, the activity following the Heidelberg discoveries led to interest in the three-body problem, and Brown (1911) extended the idea of co-orbital motion to include "horse-shoe" objects, moving past the L_4-L_3-L_5 points (see below). The first Earth co-orbital found (Wiegert *et al.*, 1997, 1998) moves on a complex horseshoe orbit with large inclination. "Quasisatellites" (Connors *et al.*, 2002, 2004) show a more subtle form of co-orbital behaviour, staying near the planet in a relative orbit resembling retrograde satellite motion. In all three cases, the semimajor axis a is very close to that of the planet, epicycles and variation of a take place approximately once per revolution, and longer-term libration makes the annual epicycle move slowly along the orbit. By Kepler's Third Law, if the semimajor axes are the same, so is the period and thus the average rate of progress in the orbit (mean motion): this is referred to as a 1:1 mean motion resonance. The libration can be correlated with small changes in a: when the small body is at slightly larger a, it falls behind in its relative motion, and vice versa.

In the late 20th and early 21st century, various classes of co-orbital companions were found in the inner Solar System. Theoretical and modelling work was also done. Specifically addressing the question of how to search for Earth Trojans, Wiegert *et al.* (2000) found that they should be stable, but in regions close to the Sun in the dawn or dusk skies. This makes ground-based searches difficult, and surveys (Whitely & Tholen, 1998; Connors *et al.*, 2000) with large-scale CCDs were not successful.

Space-Based Asteroid Searches

Observation from space removes atmospheric effects that scatter sunlight and allows the detection of wavelengths of radiation that cannot be observed from the ground. Both can help make asteroids brighter against the background sky and easier to find, even quite near the direction of the Sun. The *Wide Infrared Survey Explorer* or *WISE* (Wright *et al.*, 2010) operated in the infrared (IR), using cryogenic (dual-stage solid hydrogen) cooling, from December 2009 through 2010 September 29; after that, the mission operated without active cooling until 2011 February 1. In 2013, it was revived for passively cooled operation, detecting asteroids under the name *NEOWISE*, which also was the name of the asteroid survey conducted during the prime mission (Mainzer *et al.*, 2011a).

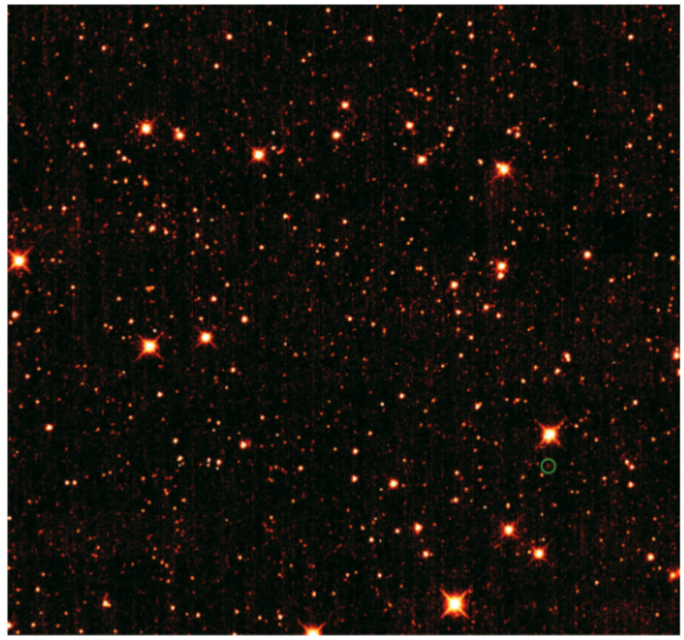

Figure 3 — WISE infrared 4.6 μm wavelength near-discovery image for 2010 TK_7. The asteroid is circled in green at the lower right. This image is centred at RA 6:14:49, Dec. -44:47:32 and is 46×46 arcmin, with south up. The bright star above the asteroid is HD 43327, roughly 10th magnitude in visible light. Faint asteroids are found as moving objects against a background of stars that are usually much brighter than they are. In this case, 2010 TK_7 was about visual magnitude 21, roughly 10,000 times fainter than HD 43327.

WISE discoveries were made public rapidly to several standard sources for information about new asteroids. The reduced orbits are available to researchers who wish to conduct further investigations.

WISE has a telescope of aperture 40 cm, comparable to mid-range amateur telescopes. Despite its modest size, the advantages of operating in space with cryogenic cooling, and detecting asteroids whose thermal emission is bright compared to the background sky, give it a detection capability literally millions of times greater than a comparably sized infrared instrument on the ground. Its mid-IR bands at 3.4, 4.6, 12, and 22 μm wavelengths (the first two of which can be used without coolant) allow detection of IR emission from asteroids, which dominates over reflected sunlight at the longest wavelengths. From a single IR band (plus knowledge of the distance from an orbital solution), asteroid diameters can be derived; the addition of visible-light observations allows determination of the albedo (reflectivity). *WISE* discovered ~150 near-Earth or potentially hazardous asteroids and 21 comets (http://neo.jpl.nasa.gov/stats/wise/, cited 2013 November 12). It detected over 158,000 asteroids, of which 34,000 were new discoveries, while it had coolant. This was fully depleted on 2010 September 29 (Mainzer *et al.*, 2012).

On 2010 October 1 (UT), shortly after the coolant ran out, *WISE* first detected 2010 TK_7. Eighteen observations were taken by *WISE* (a typical image is shown in Figure 3) and 13 more by ground-based observatories immediately following discovery, so that the orbit was reported to have a semimajor axis of 0.9991410 AU on 2010 October 7 (Minor Planet Center, 2010). *WISE* observes always at 90° from the Sun in the sky but with a large declination range. The object was at about -45° declination when discovered, and thus accessible only to Southern Hemisphere telescopes. In the visual band, the red or visual magnitude near the time of discovery was near 21, allowing determination of the asteroidal absolute magnitude, *H*, as 20.7, which for typical asteroid albedos would indicate a diameter of roughly 300 m (http://neo.jpl.nasa.gov/glossary/h.html, cited 2013 November 12). Further study of the two-band *WISE* data allowed Mainzer *et al.* (2012) to confirm this diameter as 380±120 m, with an albedo of p_V=0.06 with large uncertainty. To put the albedo in context as one of the few indicators of composition of asteroids, the study of a large proportion of the Main Belt asteroids observed with 4-band *WISE* data by Masiero *et al.* (2012) found that there is a dark population with mean albedo of 0.06 present among all asteroids, with a brighter population of mean albedo roughly 0.25 present in the inner and middle asteroid belt. Roughly one third of the near-Earth object population is dark (Mainzer, 2011b; Stuart & Binzel, 2004). 2010 TK_7 has a fairly typical albedo for dark asteroids.

The orbital geometry of 2010 TK_7 is very unfavourable for observations from Earth's surface, as shown in Figure 4. There have been few observations since the time of discovery. It spends most of its time at negative declination: the interplay of the orbital parameters with Kepler's Second Law (as illustrated in Figure 1) ensures this. To make matters worse, Figure 5 shows that the object remains faint at all times, making it visible only in large telescopes (most of which are in the northern hemisphere). Unlike almost all asteroids, it never comes to opposition (elongation of 180°), and usually is close to the Sun.

The scatter in observed magnitude near the time of discovery, shown in Figure 6, observed in the IR by *WISE* and from subsequent ground observations, indicates that the object may be elongated and rotating (perhaps fairly rapidly). With the

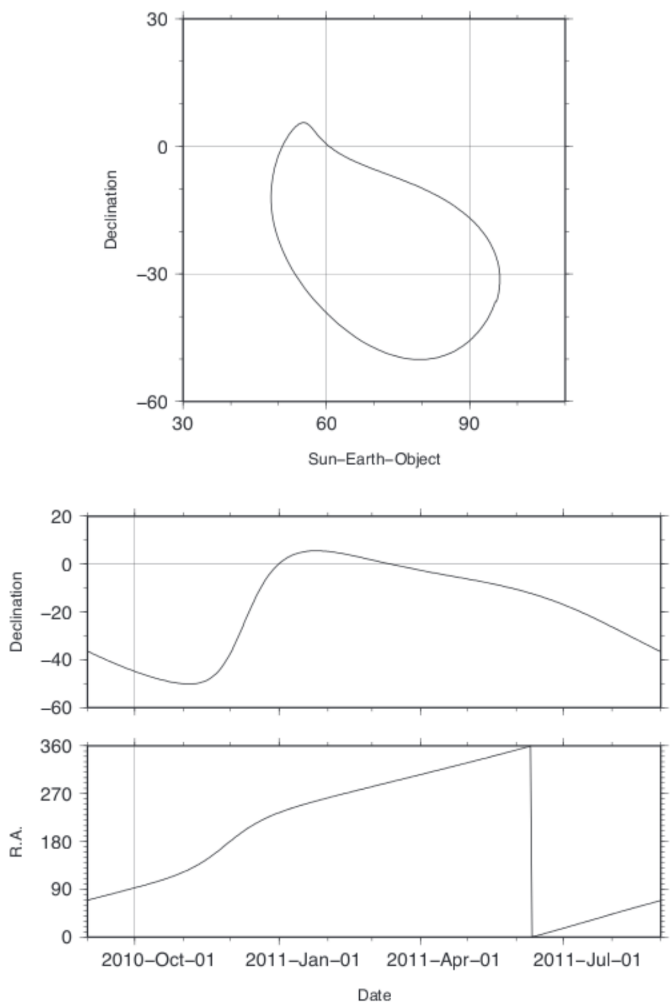

Figure 4 — Ephemeris plots for 2010 TK_7. The bottom panels show the Right Ascension (in degrees) and Declination of the asteroid from 2010 September 1 to 2011 September 1. The date of discovery, 2010 October 1, is marked by a vertical line. Note that the declination is almost always negative. The top panel combines the declination with elongation from the Sun to illustrate the combination of southerly declination and small elongation that make 2010 TK_7 a very difficult object to observe.

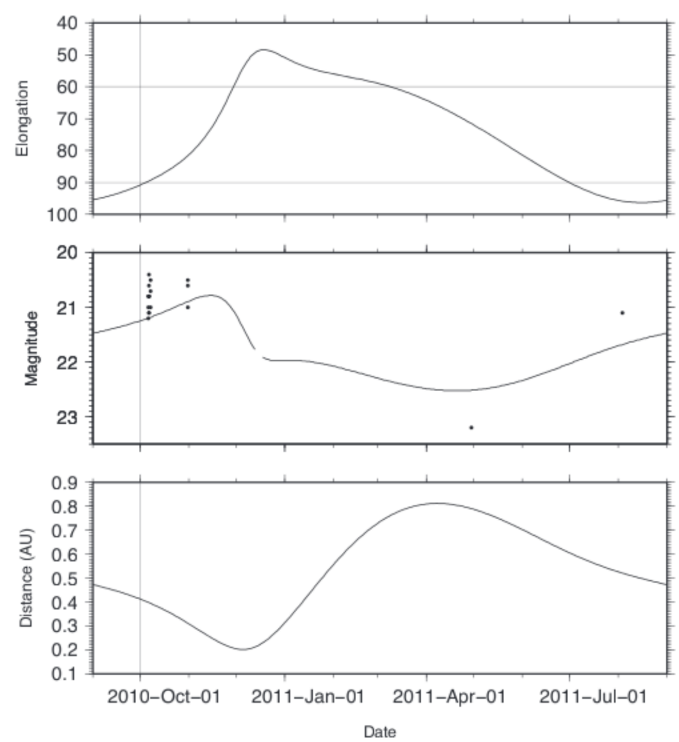

Figure 5 — Position and brightness of 2010 TK_7. The bottom panel shows the distance from Earth in astronomical units (AU). The middle panel shows the predicted optical magnitude, with dots showing observed values. The top panel (compare to Fig. 4) shows the elongation from the Sun. Horizontal lines indicate the value at the time of discovery (90°, bottom line), and the nominal Lagrangian point (60°, top line).

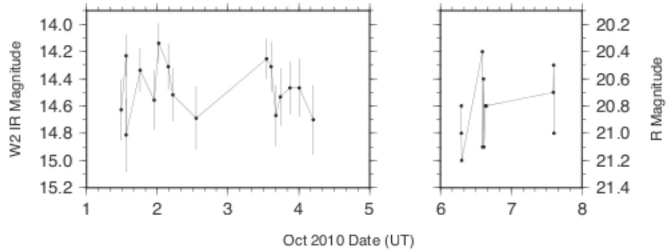

Figure 6 — W2 (4.6 µm) band IR magnitude from WISE (left panel) and ground-based R optical magnitudes (right panel) immediately following the discovery of 2010 TK_7 on 2010 October 1. The range of variation largely exceeds the errors of observation (error bars for WISE, not shown for optical but likely 0.1 to 0.3 mag), implying that the object is elongated. Thirteen optical observations are shown, but some points overlap.

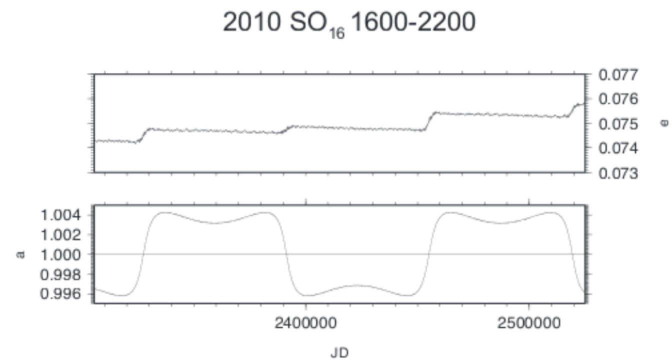

Figure 7 — Orbital parameter survey plot for 2010 SO_{16}. The bottom panel shows the characteristic "square wave" pattern of semimajor axis (a) variation for a horseshoe orbit, and the top panel the eccentricity e of the orbit. The time period is 1600 to 2200 CE, with labelling in Julian Days and axis ticks of 10,000 days.

small number of observations, a light curve or other physical information cannot be reliably determined. However, the semimajor axis of very close to 1 AU, determined quickly after its discovery, suggested that 2010 TK_7 could be of dynamical interest, since this is a characteristic of Earth co-orbital asteroids.

Discovery of the Orbital Properties of 2010 TK_7

The discussion in Section 2 above focused on the classical, geometric approach to asteroid stability near Lagrangian points, which explains Trojans well if the strict geometric conditions can be relaxed slightly, as pointed out by Charlier (1906). A more modern, and richer, approach, allowing a connection to chaos theory, emphasizes the role of resonance between asteroids and larger bodies (Lecar *et al.*, 2001). Jupiter, due to its dominant mass among planets, structures much of the asteroid belt through various resonances (including those that produce the Kirkwood gaps), and its large Trojan clouds are the premier example of 1:1 mean-motion resonance. However, 1:1 resonance with other planets, and even with asteroids (Christou and Wiegert, 2012), is possible, Section 3 mentioned Mars Trojans and objects co-orbital with Earth. Being resonant is not necessary even if *a* is very similar, and the real indication is a long-term libration of some orbital parameter. However, usually an asteroid orbit showing the same semimajor axis as a planet will be interesting and may reward investigation.

The online, sortable, Near Earth Object lists at JPL, available at the site http://neo.jpl.nasa.gov/cgi-bin/neo_elem makes checking for potential co-orbitals easy. This list, as of mid-November 2010, featured two interesting objects: 2010 SO_{16} and 2010 TK_7, both with *a* very close to 1, and both discovered by *WISE*. JPL also provides a service called "Horizons," which in its telnet form allows easy integration of orbits over several hundred years (Giorgini *et al.*, 1996). Using this, 2010 SO_{16} was found (see Figure 7) to show classic "square wave" signatures of *a* libration, with period approximately 400 years, typical of horseshoe objects (Connors *et al.*, 2002). Christou and Asher (2011) did a detailed investigation of its stability, and noted that it is fairly large among Earth co-orbitals, about 300 m in diameter. The pattern of *a* variation of 2010 TK_7, shown in Figure 8, was more like a "sawtooth" than the characteristic square wave of a horseshoe object. Three-dimensional diagrams clarify the situation, showing the relative orbit in the co-rotating frame. Unlike a standard depiction of an orbit in fixed space as seen in Figure 1, such diagrams plot the position in a frame in which the planet (in this case, Earth) is stationary. As in Figure 2, an ideal Trojan exactly at a Lagrangian point would simply be a point (in the ideal case of circular orbits). In the case of a horseshoe object, as shown for 2010 SO_{16} in Figure 9, annual motion is an approximately vertical ellipse in this frame. Over the longer-period libration, this ellipse moves around the Sun relative to Earth, tracing out a curved cylinder in three dimensions. The region near Earth is avoided, so that over a long period of time, the trajectory of the asteroid relative to Earth makes a "horseshoe." The annual motion in an ellipse is the epicycle referred to above, while the longer-term motion is libration. In contrast to the beautiful symmetry of the relative motion plot of 2010 SO_{16}, that of 2010 TK_7 is rather ugly, as shown in Figure 10. However, the asymmetry arises from staying near only one triangular Lagrangian point rather than sweeping out a horseshoe: a property of Trojan asteroids.

Some unusual aspects of 2010 TK_7 in terms of the expected observational properties of Trojan asteroids can be seen in Figure 5. Trojans are loosely expected to be near a triangular Lagrangian point, and for Earth, these are near 60° from the Sun in the sky. Yet 2010 TK_7 was discovered near 90° from the

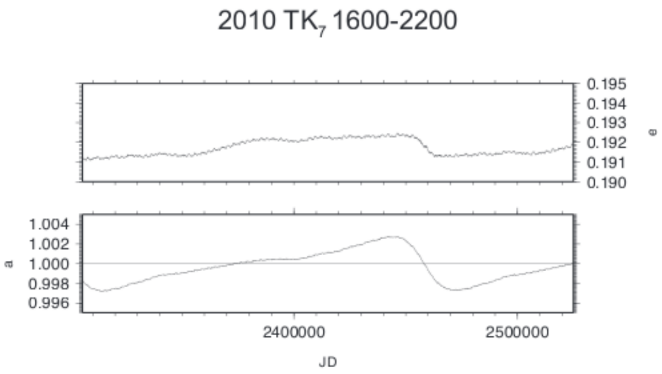

*Figure 8 — Orbital parameter survey plot for 2010 TK_7. The bottom panel shows a "sawtooth" pattern of semimajor axis (a) variation, and the top panel the eccentricity e. Labelling as in Figure. 7. An indicator of resonance in Fig. 7 and here is the variation of **a** around 1.0 AU (horizontal line).*

Sun, as dictated by the viewing geometry of the *WISE* survey satellite. After discovery, it quickly moved in to about 45° from the Sun. It spends very little time where Trojans are on average most likely to be seen (Wiegert *et al.*, 2000). Earth's Lagrangian points are very close to 1 AU away, but 2010 TK_7 near the present time never is that distant. Due to the current motion on its epicycle, and that epicycle being near Earth in the longer-term libration, 2010 TK_7 is not even near the Lagrangian point. How, then, can one know if it really is a Trojan?

Confirmation of Earth Trojan Nature

Orbits cannot be determined exactly. The last observations prior to the discovery of the likely Trojan nature of 2010 TK_7 were on 2010 October 31, and there were a total of 31 observations, including those of *WISE*. As of mid-November 2010, the best-determined value of *a*, the all-important value of the semimajor axis, was 1.00096 AU, with an uncertainty of 0.003708 AU. This may appear to be a small uncertainty, but the half-width of the resonant region for Earth is only 0.01 AU (Connors *et al.*, 2002). Since the uncertainty was nearly half the width of the resonant region, it could not be concluded with great certainty that the object was in fact an Earth Trojan. Creating "dynamical clones" by varying one or all of the orbital parameters, and performing numerical integration, gives an idea of the types of motion possible within the uncertainty range. Seven clones were created by keeping the other orbital parameters constant and varying *a* from 0.997 to 1.003 AU in steps of 0.001 AU. Integrations were performed with the Mercury integrator (Chambers 1999) and the results are shown in Figure 11. Carefully following the traces of *a* shows that three of the seven clones were Trojans, two were horseshoes, and the two at the extrema of the range, marginal horseshoes. At the time of recognition of possible Trojan properties, there was less than a 50-percent chance that the properties of the best nominal orbit were the properties of the real object. More observations were needed.

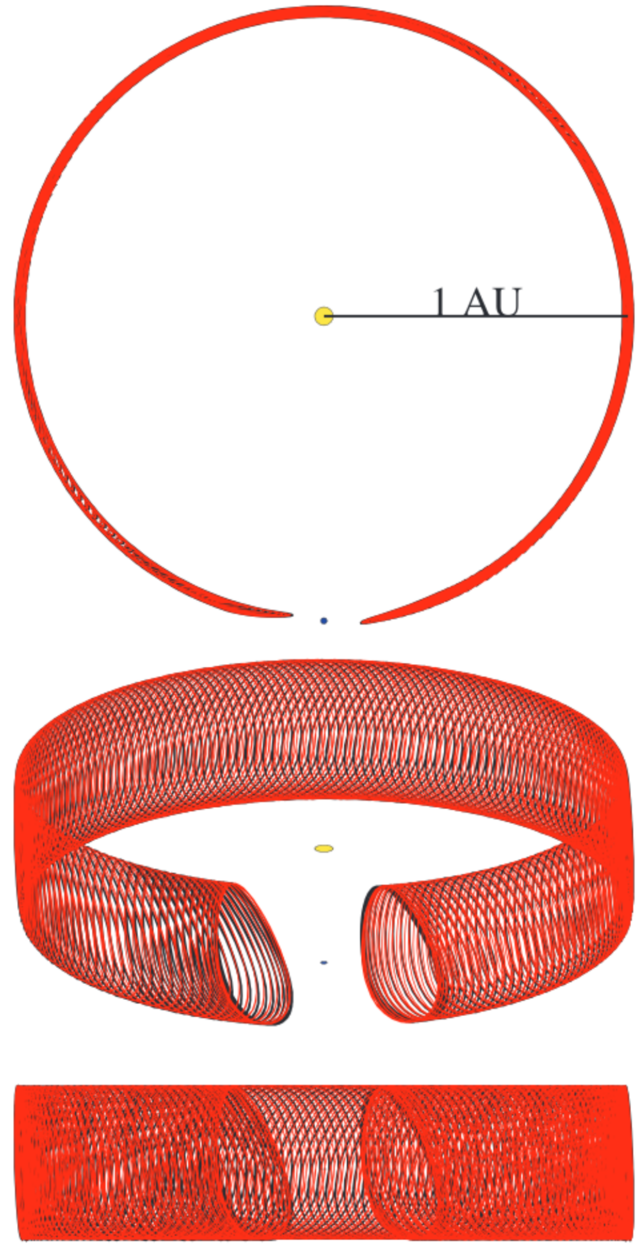

Figure 9 — Co-rotating frame visualization plot for 2010 SO_{16}. The path relative to Earth (blue dot) as it and the asteroid orbit the Sun (yellow dot) is shown from three perspectives: bottom, looking in along the ecliptic plane past Earth toward the Sun; middle, looking over Earth from 30° above the ecliptic plane; top, looking down from the north ecliptic pole. This is a classic horsehoe orbit for an object of low eccentricity and inclination.

Problematically, 2010 TK_7 was already at far-southern declination, limiting the number of telescopes available to observe it. It was dimming rapidly. Even worse, it was rapidly moving toward the Sun in the sky. It would not be observed again until an unconfirmed possible detection in early April 2011, and only on 2011 April 28 was it possible to certainly recover the object (Minor Planet Center 2011), using advanced tracking

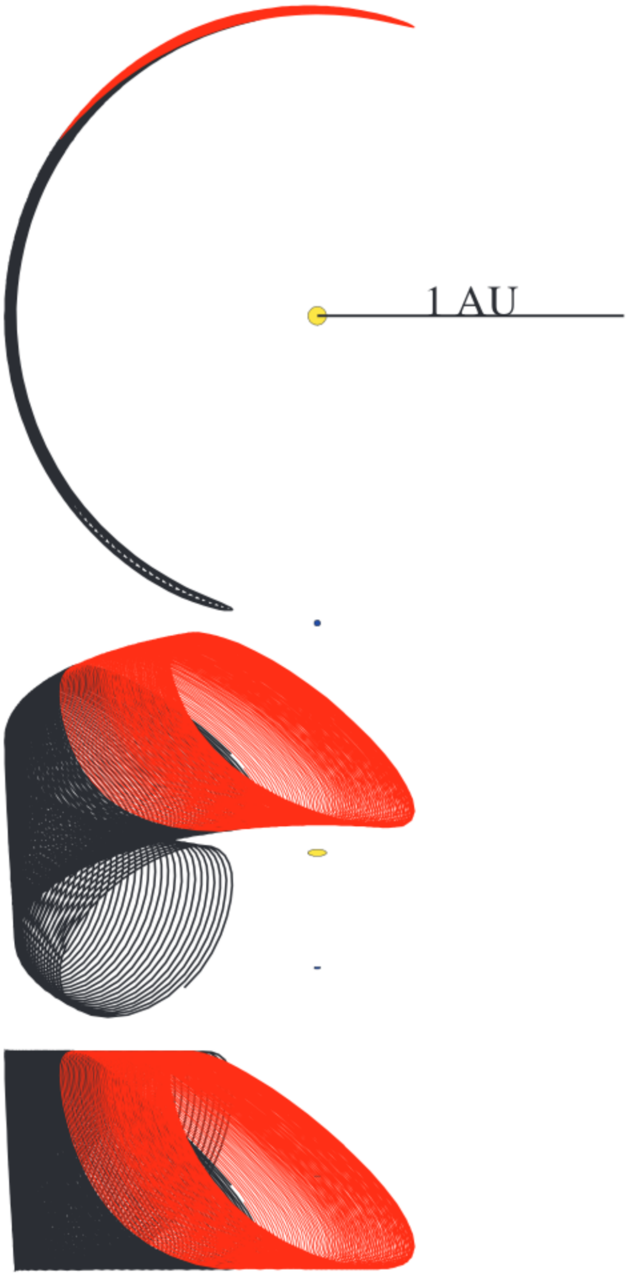

Figure 10 — Co-rotating frame visualization plot for 2010 TK$_7$. Views as in Figure 9. In this survey plot, angles are reversed compared to Figure 2. The period 1800-2000, or half a libration period, is shown, the initial half of this period in red.

techniques on the Canada-France-Hawaii Telescope. At that time, it was approximately of 23rd magnitude. Largely due to the now much-longer observational arc, the error in *a* was reduced to 2.555×10^{-5} AU. This made it virtually certain that the object was an Earth Trojan associated with the L_4 Lagrangian point, and allowed other unusual characteristics of the orbit to be discussed (Connors *et al.*, 2011). Among these was the possibility to orbit the L_3 Lagrange point unstably, a behaviour first shown as possible by Moulton (1920; p. 173).

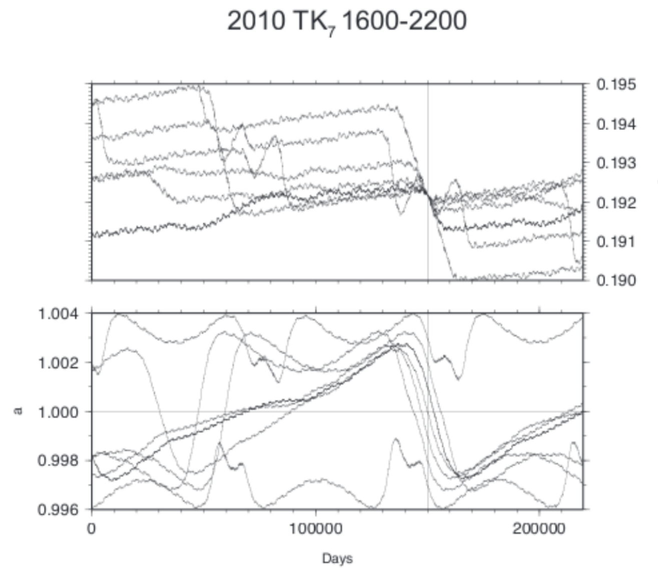

*Figure 11 — Orbital parameter plot for clones of 2010 TK$_7$. The time is labelled in days starting in AD January 1600, until 2200. The date of discovery is marked by a vertical line, and various values of the semimajor axis (bottom plot) are equally spaced on that day. They may be traced from that point to see the type of orbital motion (see text). The values of eccentricity e in the top panel are all the same on the day of discovery, since the clones were generated by varying only the semimajor axis **a**.*

The asteroid can "jump" or transition to libration about the L_5 Lagrangian point from there, in an apparently chaotic manner.

Discussion

To give context to the modern study of Trojan asteroids, we have attempted to lay out the history of celestial mechanics from a time when it was purely descriptive, up to the present age. An intervening period with a geometric and mechanistic view has given way to an era of dynamical complexity that may be studied with capable instruments, advanced computing, and developing theories of complexity and chaos (Ito & Tanikawa 2007). The first Trojan asteroid was discovered possibly in ignorance of the elegant theory of Lagrange that predicted it, which in any case that *géomètre* regarded as a purely mathematical exercise. The modern seeker of new types of behaviour in celestial mechanics has a powerful and generalizable set of tools available to guide the quest. Even so, there are new surprises around every corner.

The basic conclusions about the interesting behaviour of 2010 TK$_7$ found by Connors *et al.* (2011) were extended by Dvorak *et al.* (2012). They considered the zones of stability possible for Earth Trojans to indicate that, although 2010 TK$_7$ is an unstable temporary Trojan, there is every reason to expect others to exist. Schwartz and Dvorak (2012) examined mechanisms for temporary captures of Trojans such as 2010 TK$_7$ by planets, finding them more efficient in the inner Solar System than in the outer Solar System.

An intriguing possibility is that 2010 TK_7 is simply the "tip of the iceberg" and that other objects are deep in the Trojan zones of Earth, yet very difficult to observe. This first Earth Trojan is on an extreme orbit: this was required to even be discovered by *WISE*, which observed only at 90° from the Sun, 30° farther out than where ideal Lagrangian-point Trojans would be. If they exist, very long-lived Earth Trojans might hold material from the Earth zone of the early Solar System. Putative Earth Trojans would be relatively easy to reach with spacecraft. Stacey and Connors (2009) examined what would be required for such a mission. Although certain types of near-Earth asteroids have lower energy requirements, low-inclination Earth Trojans, if found, could still be very attractive targets. With a high inclination of nearly 21°, 2010 TK_7 itself is unlikely ever to be a rendezvous target, but spacecraft have already been to the Earth Lagrangian points several times. The *STEREO* twin spacecraft are in solar orbit to study the Sun, its outer atmosphere, and the heliosphere. By complex orbit manoeuvres involving Earth's Moon (Kaiser, 2005), they were made to orbit in opposite directions and change position by about 22° per year, thus having initially reached both triangular Lagrangian points about three years after launch on 2006 October 25.

The field of study opened by Lagrange and Wolf is now a very active and interesting one. The discovery of Earth's exotic Trojan companion holds promise that yet more surprises lurk even in our small corner of the Solar System.

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Endnotes

1 Note that in the modern era some comets are known that might be associated with Jupiter's Lagrangian points: see Jewitt *et al*. (2007)

References

Brown, E.W. (1911). On a New Family of Periodic Orbits in the Problem of Three Bodies. *MNRAS, 71*, 438-454.

Chambers, J.E. (1999). A Hybrid Symplectic Integrator that Permits Close Encounters between Massive Bodies. *MNRAS, 304*, 793-799.

Charlier, C.L. (1906). Über den Planeten 1906 TG. *AN, 171*, 213.

Charlier, C.L. (1902). *Die Mekanik des Himmels, Erster Band*. Leipzig: von Veit & Comp.

Charlier, C.L. (1907). *Die Mekanik des Himmels, Zweiter Band*. Leipzig: von Veit & Comp.

Christou, A.A. & Asher, D.J. (2011). A long-lived horseshoe companion to the Earth. *MNRAS, 414*, 2965-2969.

Christou, A.A. & Wiegert, P. (2012). A population of Main Belt Asteroids co-orbiting with Ceres and Vesta. *Icarus, 217*, 27-42.

Connors, M., Veillet, C., Wiegert, P., Innanen, K., & Mikkola, S. (2000). Initial Results of a Survey of Earth's L4 Point for Possible Earth Trojan Asteroids. *BAAS 32*, 1019-1019.

Connors, M., Chodas, P., Mikkola, S., Wiegert, P., Veillet, C., & Innanen, K. (2002). Discovery of an Asteroid and Quasi-Satellite in an Earth-Like Horseshoe Orbit. *MAPS, 37*, 1435-1441.

Connors, M., Veillet, C., Brasser, R., Wiegert, P., Chodas, P.W., Mikkola, S., & Innanen, K. (2004). Discovery of Earth's Quasi-satellite. *MAPS, 39*, 1251-1255.

Connors, M., Wiegert, P., & Veillet, C. (2011). Earth's Trojan Asteroid. *Nature 475,* 481-483, doi:10.1038/nature10233.

Cunningham, C.J., Marsden, B., & Orchiston, W. (2009). How the first dwarf planet became the asteroid Ceres. *JAHH, 12*, 240 – 248.

Cunningham, C.J., Marsden, B., & Orchiston, W. (2011). Giuseppe Piazzi: The Controversial Discovery and Loss of Ceres in 1801. *JHA, 42,* 283.

Cunningham, C.J., & Orchiston, W. (2011). Who Invented the Word Asteroid: William Herschel or Stephen Weston? *JAHH, 14*, 230-234.

Dvorak, R., Lhotka, C., & Zhou, L. (2012). The orbit of 2010 TK7: possible regions of stability for other Earth Trojan asteroids. *A&A, 541*, A127, DOI:10.1051/0004-6361/201118374.

Freiesleben, H.C. (1962). *Max Wolf - Der Bahnbrecher der Himmelsphotographie 1863-1932*, Stuttgart: Wissenschaftliche Verlagsgesellschaft.

Giorgini, J.D. *et al.* (1996). JPL's online solar system data service. *BAAS, 28*, 1158.

Greene, B. (1999). *The Elegant Universe*. New York: Vintage.

Gribbin, J. (2002). *Science: A History 1543-2001*. London: BCA.

Hills, E.H. (1914). *Address*, *MNRAS, 74*, 377-390.

Holden, E.S. (1896). The Photograpy of Planetoids, by Professor Max Wolf. *PA, 3*, 343-345.

Innanen, K. (1991). The Prediction and Discovery of a Martian Trojan Asteroid. *JRASC, 85*, 151-157.

Ito, T. & Tanikawa, K. (2007). Trends in 20th Century Celestial Mechanics. *PNAOJ, 9*, 55-112.

Jewitt, D.C., Sheppard, S., & Porco, C. (2007). Jupiter's Outer Satellites and Trojans. In F. Bagenal, T.E. Dowling, & W. McKinnon (Eds.), *Jupiter: The Planet, Satellites, and Magnetosphere*, (pp. 263-280). Cambridge, UK: Cambridge.

K.L. (unnamed contributor) (1935). Karl Vilhelm Lugvig Charlier. *MNRAS, 95,* 339-342.

Kaiser, M.L. (2005). The STEREO mission: an overview. *AdSpR., 36*, 1483-1488.

Kopff, A. (1928). Max Wolf und die Entwicklung der modernen Astronomie. *NW, 16,* 501-503.

Kuhn, T.S. (1957). *The Copernican Revolution: Planetary Astronomy in the Development of Western Thought*. Cambridge, MA: Harvard.

Lagrange, J.-L. (1772). Essai sur le problème des trois corps, Prix de l'Académie Royale des Sciences de Paris, IX; reprinted in Lagrange, J.-L. (1867-1892). In J. A. Serret & G. Darboux (Eds.), Oeuvres de Lagrange, Tome 6 (pp. 229-334). Paris: Gauthier-Villars; available at http://gallica.bnf.fr/ark:/12148/bpt6k229225j/f231, cited 2012 May 1.

Lecar, M., Franklin, F.A., Holman, M.J., & Murray, N.W. (2001). Chaos in the Solar System. *ARA&A, 39*, 581-631.

Mainzer, A. *et al.* (2011a). Preliminary Results from NEOWISE: An Enhancement to the Wide-Field Infrared Survey Explorer for Solar System Science. *ApJ, 731*, 53(13pp), doi: 10.1088/0004-637X/731/1/53.

Mainzer, A. *et al.* (2011b). NEOWISE Observations of Near-Earth Objects: Preliminary Results. *ApJ, 743*, 156(17pp), doi: 10.1088/0004-637X/743/2/156.

Mainzer A. *et al.* (2012). Physical Parameters of Asteroids Estimated from the WISE 3 Band Data and NEOWISE Post-Cryogenic Survey. *ApJ Lett., 760*, L12(6pp), doi: 10.1088/2041-8205/760/1/L12.

Masiero, J.R. *et al.* (2012). Preliminary Analysis of WISE/NEOWISE 3-Band Cryogenic and Post-cryogenic Observations of Main Belt Asteroids. *ApJ Lett., 759*, L8(5pp), doi: 10.1088/2041-8205/759/1/L8.

Minor Planet Center (2010). MPEC 2010-T45: 2010 TK7, http://www.minorplanetcenter.org/mpec/K10/K10T45.html, cited 2013 November 12.

Minor Planet Center (2011). MPEC 2010-T60: 2010 TK7, http://www.minorplanetcenter.net/mpec/K11/K11O60.html, cited 2013 November 13.

Moulton, F.R. (1920). *Periodic Orbits*, Washington: Carnegie Institution.

Moulton, F.R. (1914). *An Introduction to Celestial Mechanics*, 2nd Ed. New York: MacMillan.

Petersen, I. (1993). *Newton's Clock: Chaos in the Solar System*. New York: Freeman.

Schwartz R., & Dvorak, R. (2012). Trojan capture by terrestrial planets. *CeMDA, 113*, 23-34, DOI 10.1007/s10569-012-9404-4.

Stacey R.G., & Connors, M. (2009). Delta-v Requirements for Earth Co-orbital Rendezvous Missions. *P&SS, 57*, 822-829, doi: 10.1016/j.pss.2009.01.013.

Stuart J.S., & Binzel, R.P. (2004). Bias-corrected population, size distribution, and impact hazard for the near-Earth objects. *Icarus, 170*, 295-311.

Whitely R.J., & Tholen, D.J. (1998). A CCD Search for Lagrangian Asteroids of the Earth-Sun System. *Icarus, 136*, 154-167.

Wiegert, P., Innanen, K.A., & Mikkola, S. (2000). Earth Trojan Asteroids: A Study in Support of Observational Searches. *Icarus, 145*, 33-43.

Wiegert, P., Innanen, K.A., & Mikkola, S. (1997). An asteroidal companion to the Earth. *Nature, 387*, 685-686.

Wilson, C. (1995). The work of Lagrange in celestial mechanics. In R. Taton, & C. Wilson (Eds.), *Planetary Astronomy from the Renaissance to the rise of astrophysics: Part B: The eighteenth and nineteenth centuries* (pp. 108-130). New York: Cambridge

Wisdom, J. (1987) Urey Prize Lecture: Chaotic Dynamics in the Solar System. *Icarus, 72*, 241-275.

Wolf, M. (1906a). Photographische Aufnahmen von kleinen Planeten. *AN, 170,* 321.

Wolf, M. (1906b). Photographische Aufnahmen von kleinen Planeten. *AN, 172,* 387.

Wolf, M. (1907). Photographische Aufnahmen von kleinen Planeten. *AN, 174,* 63.

Wolf, M., & Kopff, A. (1907), Benennung von kleinen Planeten. *AN, 175,* 191.

Wright, E.L. *et al.* (2010). The Wide-field Infrared Survey Explorer (WISE): Mission Description and Initial On-Orbit Performance. *AJ, 140*, 1868-1881.