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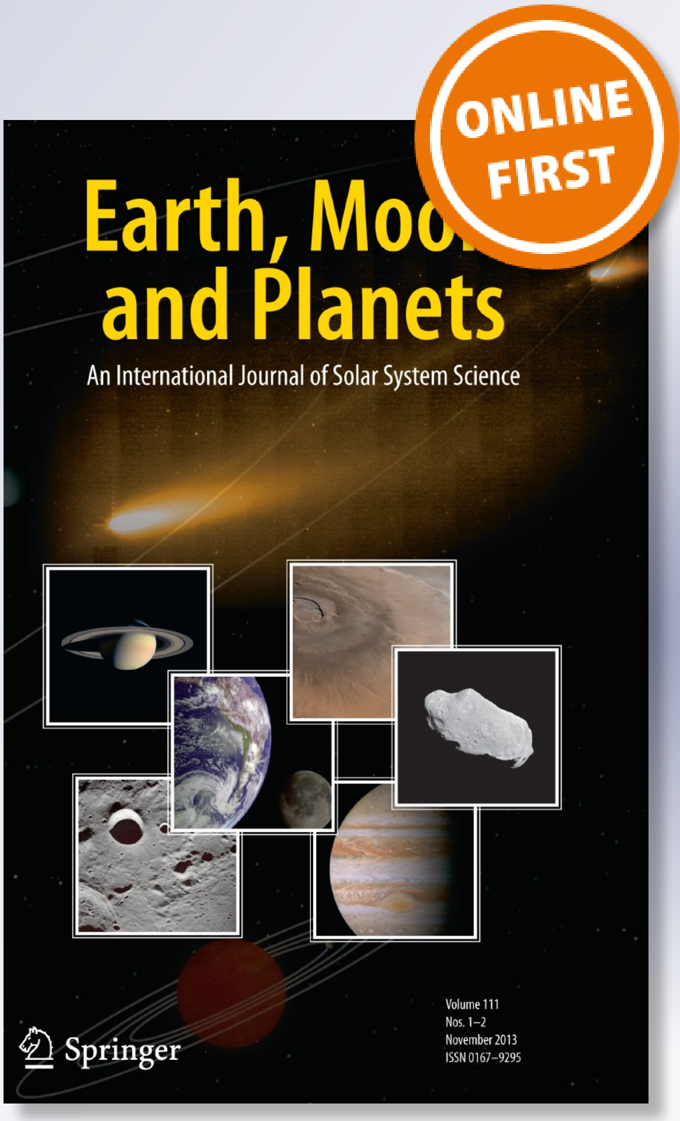

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On the Use of Meteor Camera Systems in the Detection of Kuiper Belt Objects Through Serendipitous Stellar Occultations

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Abstract The direct detection of Kuiper Belt Objects (KBOs) by telescopic imaging is not currently practical for objects much less than 100 km in diameter. However, indirect methods such as serendipitous stellar occultations might still be employed to detect these bodies. The method of serendipitous stellar occultations has been previously used with some success in detecting KBOs—Roques et al. (Astron J 132(2):819–822, 2006) detected three Trans-Neptunian objects; Schlichting et al. (Nature 462(7275):895–897, 2009) and Schlichting et al. (Astrophys J 761:150, 2012) each detected a single object in archival Hubble Space Telescope data. However, previous assessments of KBO occultation detection rates have been calculated only for telescopes—we extend this method to video camera systems, and we apply this derivation to the automated meteor camera systems currently in use at the University of Western Ontario. We find that in a typical scenario we can expect one occultation per month. However recent studies such as those of Shankman et al. (Astrophys. J. Lett. 764. doi:10.1088/2041-8205/764/1/L2, 2013) and Gladman et al. (AAS/Division for Planetary Sciences Meeting Abstracts, 2012) which indicate that the population of small KBOs may be smaller than has been assumed in the past may result in a sharp reduction of these rates. Nonetheless, a survey for KBO occultations using existing meteor camera systems may provide valuable information about the number density of KBOs.

Keywords Kuiper Belt · Solar System · Occultations

1 Introduction

The Kuiper Belt lies between 30 and 50 AU from the Sun, and the bodies found here are believed to be composed of some of the least altered material in the Solar System. The Kuiper Belt is named for Gerard Kuiper who theorized the existence of bodies beyond

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Pluto (Kuiper 1951), though he was not the first to do so: Leonard (1930) was, who was neither referenced by Kuiper (1951) nor Edgeworth (1943), who also suggested the possibility of outer Solar System bodies. The first KBO to be observed was Pluto, discovered by Clyde Tombaugh in 1930 (Aitken 1930). The next would be (15760) 1992 QB1 (Jewitt and Luu 1993). As of November 29 2012, the Minor Planet Center lists 1,299 objects known to exist between 30 and 50 AU.

The objects in the Kuiper Belt are suspected to be remnants of the Solar System's formation. By unravelling their origin and evolution, we can better constrain the processes and evolution of the early Solar System. The size distribution of the Kuiper Belt provides constraints on accretionary and collisional processes, as well as information on the mass in that region. Often, the luminosity function is used to convert magnitudes to sizes, but this is a complex problem that will not be discussed here (for an overview, see Petit et al. 2008). The currently known KBOs range in size from over 1,000 km down to below 100 km depending on the bodies' albedos [e.g. (5000) Quaoar has a diameter of 1,260 km, assuming an albedo of 0.092 (Brown and Trujillo 2004); 2003 BF_{91} has a diameter of 20 km, assuming 10 % albedo (Trilling and Bernstein 2006)].

Many early studies found the size distribution to be well described by a power law [$N(>D) \propto D^{1-q}$; where N is the number of objects with diameters greater than D, and q is the slope] (Irwin et al. 1995; Jewitt and Luu 1995; Chiang and Brown 1999), but sometimes found that a single power law slope was not enough (Irwin et al. 1995). Bernstein et al. (2004) detected fewer than expected faint objects using a single power law, implying that the luminosity function (and hence the size distribution) breaks to a shallower slope for faint objects. Fraser and Kavelaars (2009) confirmed this result, finding a large ($D > 100$ km) object power law slope of $q_1 \sim 4.8$, which turns to a slope of $q_2 \sim 1.9$ at $D \sim 60$ km. Fraser (2009) presented results of a collisional evolution model, and found that a sudden decrease in the number of objects at $r \sim$ 10–20 km is consistent with observations. Shankman et al. (2013) and Gladman et al. (2012) completed an analysis of scattered KBOs and models of their orbital distribution, and found that a divot—a decrease in the number of objects followed by a recovery—can satisfyingly explain the data. Though improvements in telescopic surveys are likely to increase our knowledge of the Kuiper Belt at smaller and smaller sizes, it will be some time before these techniques can reach the scales probed by stellar occultations.

2 Occultations

A stellar occultation occurs when an object passes through our line of sight to a distant star, causing a decrease in flux. The use of stellar occultations to detect objects that cannot be seen directly was first described by Bailey (1976) and has been discussed numerous times since (Roques and Moncuquet 2000; Roques et al. 2003; Nihei et al. 2007). The object's apparent size does not necessarily need to be larger than the star—it may appear the same size or smaller. Depending on the relative sizes of the KBO and the star, diffraction fringes (discussed in Sect. 4) may be observed (see Roques et al. 1987; Bickerton et al. 2008).

Stellar occultations allow us to probe objects that cannot be seen directly. For distant objects with atmospheres, the temperature and pressure profiles can be studied through stellar occultations (Brosch 1995; Elliot and Olkin 1996). For planets with rings, stellar occultations can allow for the discovery and study of those systems, for example: Neptune (Sicardy et al. 1991). The previous examples all refer to objects with known orbits. For large populations of small bodies, *serendipitous stellar occultations* can be used to characterize that 'invisible' population. The method of serendipitous stellar occultations is named so because it is based on the fortuitous occultation of an observed star by an unknown foreground object. Campaigns (discussed below) have been run with the intention of detecting small KBOs that cannot be seen from Earth. If this method can be successfully applied to telescopes or cameras already in use, this could provide a very cost effective method for the study of distant Solar System objects. The purpose of this work is to examine whether or not intensified video cameras already in use for meteor observation at the University of Western Ontario (see Sect. 3) might be suitable.

There have been many attempts to detect KBOs through serendipitous stellar occultations in the past (Roques et al. 2003; Bickerton et al. 2008; Lehner et al. 2006). Currently, the Transneptunian Automated Occultation Survey (TAOS II) group is working on a large scale venture. They will employ three 1.3 m telescopes in observing 10,000 stars simultaneously with the hopes of detecting small Kuiper Belt Objects (KBOs) (Lehner et al. 2012), something their previous project, the Taiwanese American Occultation Survey (TAOS), was unable to do (Bianco et al. 2010). Of the surveys that have been completed, few detections have been claimed, but the surveys themselves have allowed for the determination of upper limits on the number density of KBOs in the sky (see Bickerton et al. 2009; Schlichting et al. 2009; Bianco et al. 2009).

A single KBO occultation candidate was found at the 3σ level by Roques et al. (2003) who suggest that this may be due to Gaussian noise. Their campaign was run using the Bernard Lyot 2 m telescope of the Pic du Midi Observatory in September 2000, with data collected at 20 Hz (Roques et al. 2003). No candidates were found at the 4σ level. Roques et al. (2006) discuss their detection at the 4.2 m William Herschel telescope on La Palma of three objects in the outer Solar System, none of which lie within the Classical Kuiper Belt. An analysis of these results can be found in Bickerton et al. (2008), who claim that these events may be the result of noise. Searching from above the atmosphere, Schlichting et al. (2009) analyzed four and a half years of archival Fine Guidance Sensors data from the Hubble Space Telescope, and a single KBO occultation candidate was reported. Schlichting et al. (2012) described a new candidate found via searching 19 500 new star hours from the HSTs Fine Guidance Sensors.

These first detections are quite exciting but a larger statistically significant sample of detections will be necessary to derive the full scientific benefit of this technique. Guided by these past successes, we consider whether existing meteor camera systems—permanent installations that already run automatically when weather permits—may be able to contribute.

Various factors need to be considered prior to attempting a serendipitous KBO occultation search. To optimize a search, one should (e.g. Roques et al. 2003):

- *Observe for long times* The rarity of serendipitous occultations and the unpredictability of their time of occurrence requires long observational campaigns.
- *Look towards the ecliptic* where the Kuiper Belt is concentrated.
- *Utilize high speed photometry* KBO occultations typically last less than a second-in order to notice the flux decrease, a high speed photometer is necessary.
- *Observe 'small' stars* The smaller the apparent size of the observed star is, the smaller the occulting KBO can be. Because there are more small KBOs than large KBOs, this means a higher occultation rate.
- *Use a multi-camera system* Two nearby cameras observing the same stars should see the same occultation. Using two cameras allows for the elimination of false detections (aircraft, birds, clouds) and provides a more confident reported detection.

- *Have a deep limiting magnitude* The more stars that can be observed the likelier that an occultation will be observed.

We will see that our meteor camera system can fulfil many if not all of these requirements.

3 Project

The purpose of the work completed here is to determine the feasibility of detecting KBOs through serendipitous stellar occultations using a multi-camera system which is currently being used for the study of meteors. The camera system is run by the Meteor Physics Group at the University of Western Ontario (specifications of the system are discussed below). The cameras are already recording the sky each night (weather dependent), and may already be detecting KBO occultations though the software that searches the video for meteors does not, as currently configured, search for occultations. Occultations are unlikely to be observed in great quantity given the current set up where the cameras are pointed towards the celestial pole, where the population of KBOs is likely to be small. However a simple adjustment in pointing direction could direct them to a more favorable part of the sky. Here we attempt to determine the probable rate of detectable KBO occultations per night if the system's pointing was optimized with that in mind. At the time of this writing, work is being completed for both automated stellar photometry software (using standard aperture photometry) and hardware improvements to increase the likelihood of successful detections.

The repointing of the cameras towards the ecliptic is not without some cost in terms of meteor science. The meteor cameras are currently directed to the north in order to avoid light pollution arising from the nearby population center of London, Canada, and redirecting them to the south would degrade meteor detection. However the current meteor base stations could easily accommodate an additional camera directed to the south. So while our analysis here is based on a repointing of pre-existing cameras, our ultimate intent is to deploy new additional cameras for KBO detection so as to avoid any loss of meteor science.

3.1 Camera Systems

The Meteor Physics Group operating at the University of Western Ontario uses the Canadian Automated Meteor Observatory (CAMO) to perform optical studies on the faint meteor population. The CAMO is a multi-camera system, with one camera at the Elginfield Observatory (43°.1928N, 81°.3157W), and the other near Hickson, Ontario (43°.2642N, 80°.7721W) (Musci et al. 2012; Weryk et al. 2013). At each of the two sites there are two separate systems—a guided system and a wide-field influx system. Figure 1 shows the two systems inside the shed that houses them at the Hickson, Ontario site. The two systems have quite different specifications, and the guided system is not considered appropriate for this project. Specifications for the wide field influx system are given in Table 1. All calculations that follow are based on the wide-field system which, while watching for meteors, also happens to observe $\sim$200 stars within its field of view at a frame rate of 20 Hz every night when weather permits. It is by performing photometry of the stars in the camera's video data that we hope to detect serendipitous occultations.

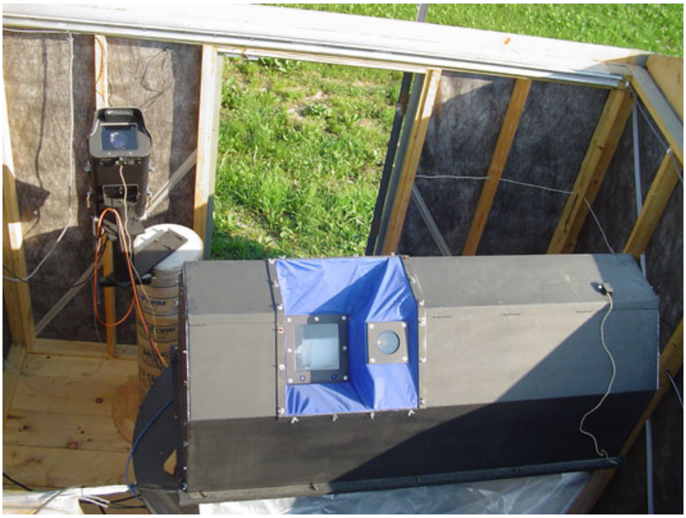

Fig. 1 In the *top left corner* is the wide field camera, and the large coffin shaped box in the *bottom right corner* is the narrow-field guided system, at the Hickson, Ontario, Canada location. The roof of the shed will roll off nightly provided the weather is appropriate, so the cameras can record meteors

Table 1 Camera specifications for the wide-field or influx camera of the CAMO

	Wide field system
Limiting stellar magnitude	6
Frame rate	20
Pixel size (arcseconds)	~70
Number of pixels	1,600 × 1,200
Field of view	21° × 21°

For more information about the cameras, see Musci et al. (2012) and Weryk et al. (2013)

4 Determining the Number of Detectable Occultations per Night

The expected number of detectable occultations is calculated based on work completed by Roques and Moncuquet (2000), generalized somewhat and modified to be applicable to our camera system. We adopt their notation for consistency, providing a number of intermediate analytical results not present in their work to aid others who might wish to assess the suitability of other systems for KBO occultation studies. Calculations were performed with Maple 15. We first determine the number of geometric occultations, and then briefly consider diffraction towards the end.

The Kuiper Belt is assumed axially symmetric and to have:

1. A spatial distribution proportional to heliocentric distance D^{-d} where we expect $d \sim 2$ (Jewitt 1999).
2. An inclination distribution proportional to $e^{-\frac{i}{H}}$, where H is the scale height.
3. A size distribution proportional to ρ^{-q}, where q is the size index and ρ is the radius. A broken-power law will be discussed later in this section.

Recent publications have suggested that 'hot' and 'cold' populations exist in the Kuiper Belt (e.g. Levison and Stern 2001; Elliot et al. 2005; Brown 2001). Thus the Kuiper Belt may be more centrally concentrated than it originally appeared, though we note that some investigators (Trujillo et al. 2001) still find this inclination distribution consistent with a single Gaussian. To simplify our calculations, we assume a single inclination distribution for the bodies in this region.

To determine the number of occultations, we first need to know the number of KBOs per unit area on the sky (to know how frequently they pass in front of stars) and the angular size they will be (to know how much of a given star they will occult, and thus whether or not they will be detectable).

The relationship between total number N of KBOs and $\nu(D, \rho, i)$, the on-sky area density of KBOs at an ecliptic latitude i at heliocentric distances D in $[D, D + \mathrm{d}D]$ and with physical radii ρ in $[\rho, \rho + \mathrm{d}\rho]$ is

$$N = \int_0^{\frac{\pi}{2}} 4\pi \cos(i) \int_{\rho_0}^{\rho_1} \int_{D_0}^{D_1} \nu(D, \rho, i)\mathrm{d}D\mathrm{d}\rho\mathrm{d}i \tag{1}$$

where D_0 and D_1 are the inner and outer limits, respectively, of the Kuiper Belt, and ρ_0 and ρ_1 are the lower and upper radii limits of the KBOs that we are considering. For the Solar System's Kuiper Belt, $D_0 \approx 30$ AU and $D_1 \approx 50$ AU. The largest KBOs have sizes $\rho_1 \approx$ 1,200 km but being rare, the precise value does not affect our results. However, the results are quite sensitive to the smallest detectable radius ρ_0 which is set by the Fresnel scale (defined below) as well as the relative angular size of the star and KBO and the signal-to-noise ratio of the measurements.

The light emitted by a star (a distant point source) will be diffracted when it hits an object. If the occulting object is large relative to the observing wavelength, and small relative to the observing distance, it is known as *Fresnel diffraction* (Born and Wolf 1965). The *Fresnel scale* is defined as

$$F = \sqrt{\frac{D_{mid}\lambda}{2}} \tag{2}$$

where D_{mid} is the distance to the object, and λ is the observing wavelength. It describes the broadening of the object's shadow (Roques 1995). Fresnel diffraction effects are important when the Fresnel scale is comparable to the size of the occulting object. We chose a mid-range value from the spectral response of the intensifier used with our camera system ($\lambda = 700\ nm$) and find that the Fresnel scale is 1.45 km.

Given the assumptions above, and assuming the distributions in D, ρ and i are independent, then $\nu(D, \rho, i)$ obeys

$$\nu(D, \rho, i) = \bar{C_0}\rho^{-q}D^{-d}e^{-\frac{i}{H}} \tag{3}$$

where $\bar{C_0}$ is a constant that needs to be solved for using Eq. 1. The integral becomes

$$N = \int_0^{\frac{\pi}{2}} 4\pi\cos(i) \int_{\rho_0}^{\rho_1} \int_{D_0}^{D_1} \bar{C_0}\rho^{-q}D^{-d}e^{-\frac{i}{H}}\mathrm{d}D\mathrm{d}\rho\mathrm{d}i$$

which, for $q \neq 1$ and $d \neq 1$ is

$$N = \frac{4\bar{C_0}\pi(\rho_1^{1-q} - \rho_0^{1-q})(D_1^{1-d} - D_0^{1-d})H(1 + He^{-\frac{\pi}{2H}})}{(1-q)(1-d)(1+H^2)} \tag{4}$$

If $q \gg 1$ which we expect to be the case in the Kuiper Belt, then Eq. 4 reduces to

$$N \approx \frac{4\bar{C_0}\pi\rho_0^{1-q}(D_1^{1-d} - D_0^{1-d})}{(q-1)(1-d)F(H)} \tag{5}$$

where for convenience we have defined

$$F(H) \equiv \frac{(1+H^2)}{H(1+He^{-\frac{\pi}{2H}})} \tag{6}$$

This gives us the needed expression

$$\bar{C_0} \approx \frac{N(q-1)(1-d)F(H)}{4\pi\rho_0^{1-q}(D_1^{1-d} - D_0^{1-d})} \tag{7}$$

In order to keep the dependence on N apparent in subsequent calculations, we define the related constant $C_0 \equiv \bar{C_0}/N$

$$C_0 \approx \frac{(q-1)(1-d)F(H)}{4\pi\rho_0^{1-q}(D_1^{1-d} - D_0^{1-d})} \tag{8}$$

Our intention is to use v to determine $\delta(\varphi, i)$ the on-sky density of KBOs at a given latitude i as a function of their angular size φ. Using the apparent radius φ rather than physical radius ρ requires changing variables in Eq. 3. The small angle formula $\varphi = \frac{\rho}{D}$ is the required relation, but we include a constant $C_1 = 1.495 \times 10^8$ km/AU to allow ρ to be specified in kilometers while D has units of AU, and obtain φ in units of radians: $\varphi = \frac{\rho}{C_1 D}$. Since v is the density over a interval $d\rho$, we also need to convert $\mathrm{d}\rho$ to $\mathrm{d}\varphi$. The usual Jacobian allows us to determine that $\mathrm{d}\rho = C_1 D \mathrm{d}\varphi$.

$$\delta(\varphi, i) = \int_{D_0}^{D_1} v(D, C_1\varphi D, i) C_1 D \mathrm{d}D \tag{9}$$

$$= \int_{D_0}^{D_1} N C_0 C_1^{1-q} \varphi^{-q} D^{1-q-d} e^{-i/H} \mathrm{d}D \tag{10}$$

Integrating yields (assuming $q + d - 2 \neq 0$)

$$\delta(\varphi, i) = \frac{N C_0 C_1^{1-q}(D_0^{-q-d+2} - D_1^{-q-d+2})}{q+d-2} \varphi^{-q} e^{-i/H} \tag{11}$$

or

$$\delta(\varphi, i) = N C_2 \varphi^{-q} e^{-i/H} \tag{12}$$

where we again collect terms for convenience into a single constant

$$C_2 \equiv \frac{C_0 C_1^{1-q}(D_0^{-q-d+2} - D_1^{-q-d+2})}{q+d-2} \tag{13}$$

Equation 12 gives us an expression for $\delta(\varphi, i)$, the number density of KBOs per steradian of the sky in the angular size interval $[\varphi, \varphi + d\varphi]$.

Using our default values given in Table 2, we get a value of δ which agrees within a factor of two with the value of Roques and Moncuquet (2000) when allowance is made for the fact that their value is a number density per milliarcsecond squared of sky, and their units for φ are also milliarcseconds.

4.1 Detection Rates

In a random star field, most stars we detect will be just above the limiting magnitude m_{lim}. If R_* is the physical radius of the star, and D_* the distance it would have to be at to have an apparent magnitude at our detection limit, the apparent radius of the star will be

$$\Phi_* \approx \frac{R_*}{D_*} = \frac{R_*}{10 \text{ parsecs } 10^{(m_{\text{lim}}-M)/5}} \tag{14}$$

where M is the absolute magnitude of the star. The meteor cameras we examine here are red-sensitive and so we take an F5 star (assuming a physical radius of 1.2 solar radii) as our reference. Not all of the approximately 1,750 stars with limiting magnitudes less than or equal to 6 that are within 20° of the ecliptic (SIMBAD, http://simbad.u-strasbg.fr/simbad/) are F5 stars but we choose this spectral class for consistency with Roques and Moncuquet (2000) (who examined O5, F5 and M5 stars) and as being the most typical of these three classes.

From Eq. 14, at a limiting magnitude of 6 such a star has an angular radius of 5.8×10^{-10} radians or 0.1 ms, equivalent to a projected radius of 3.5 km at 40 AU. This is larger than the Fresnel scale (1.4 km) and so we expect that most occultations will involve KBOs whose angular diameter is smaller than the stars ($\varphi < \Phi_*$).

Now that we have the on-sky density of KBOs at different angular sizes, we need to calculate the probability that a KBO occults a star during a given time. The angular area (in steradians) a KBO covers as it travels across the sky in Δt is approximately $2\varphi v_n \Delta t$, where

Table 2 Values used in calculation of the rate of KBO occultations

Variable	Unit	Value	Reason/reference
q		4	Slope of KBO differential size distribution (Jewitt 1999)
d		2	Slope of spatial distribution
H	rad	$\frac{\pi}{6}$	Jewitt et al. (1996)
D_0	AU	30	~Inner boundary of KB
D_{mid}	AU	40	~Middle of KB
D_1	AU	50	~Outer boundary of KB
N		10^{11}	Number of Kuiper Belt Objects larger than 1 km (Jewitt 1999)
ρ_0	km	1	~Minimum radius of detectable KBO (Fresnel scale)
ρ_1	km	1000	~Maximum radius of detectable KBO
i	rad	0	Ecliptic latitude (towards ecliptic)
ω	rad	0	Direction of observation (towards opposition)
Φ_*	rad	6×10^{-10}	F5 star at app. magnitude 6
Δt	sec	10,800	One night: 3 h
C_1	$\frac{\text{km}}{\text{AU}}$	1.4958×10^{8}	Conversion factor

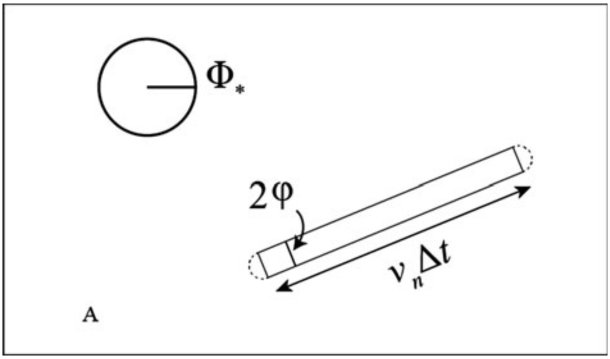

Fig. 2 Determining the probability that a KBO will occult a star. In time Δt, a KBO with angular radius φ, and angular velocity v_n will travel a distance $v_n \Delta t$, and will cover an area $2\varphi v_n \Delta t$. *Note* Φ_* is the angular radius of the star

φ is the KBOs angular radius and v_n is its angular velocity projected onto the plane of the sky. The geometry is shown in Fig. 2.

The probability that this object occults a star located on a patch of sky covering area A is

$$P_{occ} = \frac{2(\Phi_* + \varphi) v_n \Delta t}{A} \tag{15}$$

where the factor of φ has been replaced with $\varphi + \Phi_*$ because as noted in Roques and Moncuquet (2000), an occultation occurs when the minimum distance between two objects is less than the sum of their radii. Here we will consider only events where the KBO (which is expected to be smaller than the star) goes entirely in front of the star, which have the probability

$$P_{full} = \frac{2(\Phi_* - \varphi) v_n \Delta t}{A} \tag{16}$$

The angular velocity of the KBOs v_n depends on the direction the observations are taken in. The linear velocity of a KBO v_0 is

$$v_0 = v_\oplus \left(\cos \omega - \frac{1}{\sqrt{D_{mid}}} \right) \tag{17}$$

where $v_\oplus$ is the orbital speed of Earth, ω is the angle between our line of sight to the KBO and the anti-solar direction, and D_{mid} is a typical heliocentric distance of a KBO (taken here to be 40 AU). Observing at opposition increases the occultation rate while shortening their duration.

From Eq. 16, the expected number of occultations per star at ecliptic latitude i by KBOs with angular sizes from $[\varphi, \varphi + d\varphi]$ is then

$$n_{occ}(\varphi, i) = \frac{2(\Phi_* - \varphi) v_n \Delta t N}{A} \tag{18}$$

Noting that the density of KBOs of angular radius φ on the sky plane is $\delta = \frac{N}{A}$, we may rewrite Eq. 18 as

$$n_{occ}(\varphi, i) = 2\delta(\varphi, i)(\Phi_* - \varphi) v_n \Delta t \tag{19}$$

or substituting in our expression for δ (Eq. 12)

$$n_{occ}(\varphi, i) = 2NC_2 v_n \Delta t e^{-i/H} \varphi^{-q} (\Phi_* - \varphi) \tag{20}$$

In order to calculate the total number of occultations per time Δt we must integrate Eq. 19 over the detectable radii of KBOs

$$N_{occ}(i) = \int_{\varphi_0}^{\varphi_1} n_{occ}(\varphi, i) \mathrm{d}\varphi \tag{21}$$

In the integration limits of Eq. 21, φ_0 is the smallest angular radius of a KBO we expect to see and is dependent upon both the size of the star being observed and the noise levels in our detector. The expression used to calculate φ_0 is discussed later in this chapter. On the other hand, φ_1 is the largest detectable angular radius of a KBO. Because large KBOs are much less numerous than small ones, the value of φ_1 is not crucial, or one that we are sensitive to.

Substituting Eqs. 20 into 21 yields

$$N_{occ}(i) = 2NC_2 v_n \Delta t e^{-i/H} \int_{\varphi_0}^{\varphi_1} \varphi^{-q} (\Phi_* - \varphi) \mathrm{d}\varphi \tag{22}$$

The integral with respect to φ has the general form (assuming $q \neq 1,2$)

$$\int \varphi^{-q} (\Phi_* - \varphi) \mathrm{d}\varphi = \frac{\varphi^{1-q} [(2-q)\Phi_* - (1-q)\varphi]}{(q-1)(q-2)} \tag{23}$$

which if $q \gg 0$

$$\int_{\varphi_0}^{\varphi_1} \varphi^{-q} (\Phi_* - \varphi) \mathrm{d}\varphi \approx \frac{\varphi_0^{1-q} [(2-q)\Phi_* - (1-q)\varphi_0]}{(q-1)(q-2)} \tag{24}$$

Putting all our results together, we can derive a final expression for the number of occultations expected of a single star with angular radius Φ_* per unit time.

$$N_{occ}(i) \approx \frac{2NC_2}{(q-1)(q-2)} v_n \Delta t e^{-i/H} \varphi_0^{1-q} [(2-q)\Phi_* - (1-q)\varphi_0] \tag{25}$$

Here angular quantities are all in radians, v_n is in radians per seconds and Δt is in seconds.

To complete our calculation of the expected number of detectable geometric KBO occultations, we need a few final values. Here we take $\Delta t = 3$ h, assuming a typical night of partly clear observations, and take $i = 0$ pointing towards the ecliptic plane, where the Kuiper Belt is concentrated. We note that pointing the cameras towards the ecliptic plane is a function of season, however re-pointing the cameras should not be an issue—the cameras are currently pointing towards $\sim 32°$ from the zenith, and to be pointing towards the ecliptic, on average during the year, we need to be pointing towards $\sim 43°$ from the zenith. We assume that the Kuiper Belt has 10^5 objects down to $\rho = 100$ km (Jewitt, 1999) and—for the moment—that this slope ($q = 4$) continues down to at least 1 km. If the differential size distribution is proportional to D^{-q} then the cumulative size distribution should be proportional to D^{1-q}. The cumulative size distribution $N(>\rho)$ is the number of bodies with radii larger than ρ. The power-law implies that

$$N(>\rho) = N_0 \rho^{1-q} \tag{26}$$

where N_0 is a constant determined from $N(>100\ \text{km}) = 10^5$ to be $N_0 = 10^{11}$ if $q = 4$. From Eq. 26 the cumulative number of KBOs down to $\rho = 1$ km is $N(>1\ \text{km}) = N_0\ 1^{1-q} = 10^{11}$ as Roques and Moncuquet (2000) found. If however the slope is not constant down to such sizes as evidence is starting to show (Shankman et al. 2013; Gladman et al. 2012), N should be reduced. The question of the number of KBOs is returned to below.

If σ is the rms noise as a fraction of the measured star's brightness and we wish a detection at n times σ, then the KBO must occult enough of the star such that

$$\frac{\pi\varphi^2}{\pi\Phi_*^2} \geq n\sigma$$

which translates into an expression for the smallest angular size φ_0 detectable

$$\varphi_0 = \Phi_* \sqrt{n\sigma} \tag{27}$$

The fluctuation in a star's light caused by a KBO occultation would be easy to observe if the detectors were free of noise, but that is not the case. Random fluctuations in the amount of light received due to the atmosphere and other sources as well as intrinsic noise in the detector might create a flux reduction that could be mistaken for an occultation. To evaluate φ_0, video data from the CAMO is assessed for its noise characteristics.

The system in question (the 'influx system' in Weryk et al. 2013) is comprised of two identical camera systems separated by 45 km. These image intensified cameras run at 20 frames per second, and collect data which is then analyzed in search of meteors. Musci et al. (2012) provides a detailed description of the system. With 173 frames of data recorded June 11 2012 beginning at 2:51:07 UTC, using the program METeor AnaLysis (METAL) (Weryk and Brown 2012), the rms signal fluctuation was determined. Six stars, each of a different apparent magnitude from 4 to 6 were selected in each subsequent frame. The signal fluctuation was calculated for each of the six stars using the standard deviation and average of the apparent magnitude. The relative average of these six values was then calculated to be 4 %: $\sigma \approx 0.04$. Inserting these values into Eq. 27 gives a smallest detectable apparent radius $\varphi_0 = 0.05$ mas for a 4 σ detection. This φ_0 value is equivalent to a 1.4 km radius object seen at 40 AU.

The total number of occultations is also dependent upon the number of stars visible to the camera, which is dependent both on the number of stars in the sky brighter than the camera's limiting magnitude and camera's field of view. Frames from the CAMO system were run through Source Extractor (Bertin and Arnouts 1996) to determine the number of stars visible above certain thresholds. A maximum stacked image (consisting of over 100 frames and created by taking the maximum pixel value from all the frames used) is shown in Fig. 3. Single frames (not stacked frames as shown in Fig. 3) were run through Source Extractor, which determined that at the 4σ level, we can see $\sim$200 stars with the wide-field camera on a single frame.

To calculate the number of geometric occultations per night that the camera can detect, we need to substitute the values just discussed into Eq. 25, and multiply by the number of visible stars which depends on the pointing direction but is $\sim$200. The camera is currently typically pointed at a zenith angle of 32° which we take as representative of the average zenith angle of the ecliptic (43°) from the site in question.

$$N_{occ,total} = N_{occ} \cdot Number\ of\ stars \tag{28}$$

Fig. 3 Maximum stacked image from the wide-field system at Elginfield. A meteor trail is seen in the upper middle portion. Source Extractor provided 'star counts' at various threshold levels

$$= 1.75 \times 10^{-4} \frac{\text{Occultations}}{\text{Star} \cdot 3\ \text{h}} \cdot 200\ \text{stars} \tag{29}$$

We find that we should detect 0.035 occultations per night, which is equivalently one occultation per 28.6 nights.

The duration of the occultation is the time it takes for a KBO to cross the projected size of the star at a linear velocity v_0: at 25 km/s, the shadow will cross the detector in 0.14 s, corresponding to a minimum frame rate 7.1 Hz in this case. The detector need not necessarily operate at such high frame rates as the occultation may remain detectable even if sampled at a lower rate but the CAMO system operates at 20 Hz, and so it should be sampling fast enough capture the occultations in several frames.

We now discuss and apply diffraction effects to our derivation. When considering serendipitous stellar occultations by KBOs, diffraction should be taken into account.

Could we detect KBOs with radii smaller than the 1 km we assumed earlier? This hinges on going beyond geometric occultations into the diffraction regime. As Roques and Moncuquet (2000) note, diffraction effectively broadens the cross-section of KBOs, though the resulting signal becomes one comprised primarily of diffraction 'ripples' rather than a sharp dip in stellar brightness. Using Fig. 3a–e from Roques and Moncuquet (2000), we find that at our signal fluctuation level (0.04) we could see objects as small as 0.1 times the Fresnel scale for most stars. Supposing we can actually detect objects as small as $\rho_0 = 0.2$ km, we will see 5^{1-q} $\sim$125 times more occulting bodies, yielding a total of a few detectable occultations per night. We note however that we expect our detected KBOs to be significantly smaller than the stars they occult which will smooth out the diffraction fringes to a certain extent.

Detecting diffraction fringes from occultation events depends on being able to dig signals out of low signal-to-noise data, a capability not yet demonstrated with the CAMO system. Though the intensified camera systems provide high-speed photometry, their noise properties are relatively poor when compared to traditional CCDs. Not only is the noise non-Gaussian in general, but our initial tests show short time-scale ($\sim$minutes) variations in the noise levels. Whether these effects can be removed through suitable flat-fielding or other image processing techniques, or whether software processing (such as matched filtering, where one convolves the data with an expected 'template signal' to search for the signal) might make searches for diffraction signals practicable is as yet unclear and will be addressed in future work.

The number of expected occultation events depends strongly on the population of the Kuiper Belt itself. If the slope of the size distribution is a broken power law breaking to a shallower slope, the analysis here still holds in the limit that the break in the slope occurs at sizes large enough that the bulk of the KBOs are described by this new slope. This is perhaps likely given that small objects typically vastly outnumber large ones in the solar system. Such a situation would however require an adjustment to N in our calculation, where the total number of KBOs would have to be determined based on this new size distribution. A break to a differential slope of $q = 3$ at 100 km would mean that the number of KBOs at 1 km in Eq. 26 is approximately 100 times fewer than calculated here, and the occultation rates of Eq. 29 must be scaled accordingly. If this were the case, serendipitous detection of KBOs may not prove practical with the technique described here. However, current indications are that while there may be a drop or "divot" in KBO numbers, this drop is followed by a recovery at smaller sizes (Shankman et al. 2013; Gladman et al. 2012), though these studies put the number of KBOs down to roughly 1 km sizes ($H = 18$) at only 2×10^9, which would reduce our detection rates by a factor of fifty.

The possibility also exists that the system will not detect any occultation events. If this is the case, the non-detection still provides an upper limit on the size distribution of small KBOs, and points to a shallowing of the slope q and a smaller than expected Kuiper Belt at these sizes. Regardless of predictions based on observations taken at large sizes, the question of whether or not km size KBOs exist in abundance can only be answered by actually making the observations. The hopeful rate of geometric occultation detections revealed in Eq. 29 means that meteor camera techniques may provide a viable method for determining the properties of KBOs that are inaccessible to telescopic observations.

5 Conclusion

Taking into consideration the expected properties of the Kuiper Belt and the features of the intensified video cameras, we determined the rate at which we can expect to detect serendipitous stellar occultations by KBOs with existing meteor systems. The rates are strongly dependent on the properties of the KBOs, particularly the number of objects at km sizes. Rates might range as high as a few per month under persistent clear skies, though they may also be two or three orders of magnitude lower depending on the true population of the Kuiper Belt. The advantages of using existing meteor camera systems partially mitigate the uncertainties involved. Since the systems already exist and are taking data, the relative investment required to analyze the data is small compared to that needed to construct, test and operate a system solely for the purpose of occultation detection. Even the addition of separate occultation cameras to the systems constitute modest investments compared to starting from scratch. The camera systems currently in operation at the University of Western Ontario are expected to operate for decades, with continually improving characteristics as technology advances. Whether KBOs are detected by these or not, the results from such a search may provide important constraints on the number density and size distribution of the Kuiper Belt.

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