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Transient co-orbital asteroids

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Abstract

We analyze the orbital behavior of four new co-orbital NEOs and the Earth horseshoe object 2002 AA29. The new objects are 2001 CK32, a 3753 *Cruithne*-like co-orbital of Venus, 2001 GO2 and 2003 YN107, two objects with motion similar to 2002 AA29. 2001 CK32 is on a compound orbit. The asteroid reverses its path when the mean longitude difference is -50°. Its motion is chaotic. 2001 GO2 is an Earth HS orbiter with repeated transitions to the QS phase, the next occurring 200 years from now. The HS libration period is 190 years and the QS phases last 45 years. For 2002 AA29, our simulations permit us to find useful theoretical insights into the HS–QS transitions. Its orbit can be simulated with adequate accuracy for 4400 years into the future and 1483 years into the past. The new co-orbital 2003 YN107 is at present an Earth QS. It has entered this phase in 1997 and will leave it again in 2006, completing one QS cycle. Like 2002 AA29, it has frequent transitions between HS and QS. One HS cycle takes 133 years.

Keywords: Asteroid; Dynamics; Earth

1. Introduction

Co-orbital motion, which is a common phenomenon in the Solar System, manifests itself in three types of motion: the classical tadpole, the horseshoe (HS) and the quasi-satellite (QS) (Jackson, 1913; Mikkola and Innanen, 1997). The latter orbits are retrograde satellite orbits with large heliocentric eccentricity and are well known in the context of the three-body problem. Such orbits are stable with increasing relative orbital energy (Hénon, 1969). The three orbit-types can be distinguished through the librational properties of the mean longitude difference between the asteroid and the planet. The latter is defined as $\phi = \lambda - \lambda_p$, where the subscript is used to denote planetary quantities.

Tadpole orbits librate about $\phi = \pm 60^\circ$ depending on whether they are leading ($+60^\circ$) or trailing the planet (-60°). Horseshoe orbits librate around $\phi = 180^\circ$ and quasi-satellite orbits librate around $\phi = 0^\circ$. For orbits with large enough eccentricity and/or high enough inclination, transitions between these orbit types are possible (Namouni, 1999) as well as the formation of so-called 'compound orbits' (Namouni et al., 1999). Co-orbital asteroids in tadpole motion are termed Trojans. Today, more than a thousand jovian Trojans are known, several martian, and one neptunian. The saturnian system contains tadpole orbiters as well as the horseshoe companions Janus and Epimethius (see, e.g., Dermott and Murray, 1981). It is conjectured that the majority of these are primordial objects (see, e.g., Brasser et al., 2004; Mikkola et al., 1994; Tabachnik and Evans, 2000). The Earth has four co-orbital companions: 3753 *Cruithne* (Wiegert et al., 1997), 2002 AA29 (Connors et al., 2002), 2000 PH5 (Wiegert et al., 2002; Margot and Nicholson, 2003), 2001 GO2 (Wiegert et al., 2002) and one QS object that will soon revert back to HS (Connors et al., 2004). Venus has one co-orbital object,

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Table 1
Orbital elements of co-orbital asteroids in the inner Solar System. The three known Mars Trojans are not included in this list. (Epoch = JD 2453000.5; J2000.0 ecliptic and equinox.)

Object	a (AU)	e	i (°)	Ω (°)	ω (°)	M (°)
2001 CK32	0.725403	0.382579	8.137	109.579	234.137	18.719
2002 VE68	0.723648	0.410496	8.979	231.672	355.527	119.013
3753 *Cruithne*	0.997703	0.514861	19.811	126.312	43.714	352.38
2000 PH5	1.00061	0.230133	1.712	280.256	275.784	231.841
2001 GO2	1.00559	0.167959	4.616	193.634	265.125	331.354
2002 AA29	0.997556	0.012954	10.746	106.688	96.288	256.669
2003 YN107	0.997211	0.023485	4.381	269.663	111.681	71.661

2002 VE68, which is also the first known QS in action at present (Mikkola et al., 2004). Christou (2000) has studied the behavior of probable Earth and Venus co-orbitals and found that 10563 *Izdhubar*, 3362 *Khufu* and 1994 WF2 may become Earth co-orbitals in the future while 1989 VA may become a Venus co-orbital.

Morais and Morbidelli (2002) have obtained the size and orbital distributions of Near-Earth Asteroids (NEAs) that are expected to be co-orbitals of the Earth in a steady-state scenario. They predict 0.65 objects with $H < 18$ and 16 with $H < 22$ and conclude that these objects are not easily observed as they are distributed over a large area in the sky and spend most of the time away from opposition where they may be too faint.

The presentation of this paper is divided as follows: in Section 2, the method used for our numerical orbit computations is outlined. Section 3 is devoted to a summary of the motion of 3753 *Cruithne*, 2002 VE68 and the temporary Earth HS object 2000 PH5. In Section 4, the motion of 2001 CK32, a new Venus co-orbital with motion similar to that of 3753 *Cruithne* is described. Section 5 deals with the orbit of 2002 AA29, Section 6 with 2001 GO2, and Section 7 with the Earth QS 2003 YN107. In Section 8 the summary and conclusions are provided.

2. Numerical experiments

The orbital elements for all objects were taken from the NeoDys[1] pages. From the covariance matrix, a further 49 clone orbits were generated to take uncertainties in the elements into account. The variations in the element vector $\mathbf{q}$ were computed using $\delta\mathbf{q} = \sum_{k=1}^{6} \xi_k \sqrt{\lambda_k}\mathbf{X}_k$, where λ_k and $\mathbf{X}_k$ are, correspondingly, the eigenvalues and eigenvectors of the covariance matrix $\mathbf{C}$, while ξ_k are random numbers with a nearly Gaussian distribution and $\bar{\xi}_k^2 = 1$ (see Brasser et al., 2004). The orbit computations were done using a Wisdom–Holman algorithm (Wisdom and Holman, 1991) with a timestep of 0.5 days (0.05 days for 2002 AA29) for 20 000 years into the future and, where necessary, into the past.

[1] http://newton.dm.unipi.it/neodys/.

3. Overview

We performed a search of those asteroids whose semimajor axis is in an annulus $|a - a_{\mathrm{p}}| < \varepsilon$ where $\varepsilon = (1/3\mu)^{1/3}$, with μ the planetary mass in solar masses. We simulated these objects and their clone orbits and determined which ones showed co-orbital behavior. The resulting objects have their elements listed in Table 1. These objects are transient co-orbitals: they are injected, through some mechanism, into the co-orbital zone of one of the inner planets, remain there for awhile and then leave. This implies there is a quasi-steady-state flux of these objects moving in and out of the co-orbital region, according to Christou (2000). In this paper, we describe known other transient co-orbital NEOs and focus on four asteroids: 2001 CK32, 2002 AA29, 2001 GO2 and 2003 YN107.

The sensitivity of the orbit to initial values was studied by using the tangent map (Mikkola and Innanen, 1999) to determine the Lyapunov times of the trajectories. As usual, the trajectories of all these asteroids turn out to be chaotic, with the Lyapunov times generally ~ 100 years.

Tabachnik and Evans (2000) showed the dynamics of Venus and Earth co-orbitals is very similar, so that it is likely that Venus has co-orbital objects with similar properties to those described here for the Earth.

3.1. 2002 VE68

This object is currently a Venus QS and the first such object seen in action. It is also the first discovered Venus co-orbital. A detailed discussion of its orbit is given in Mikkola et al. (2004). Its arc length is only 24 days so that its orbit not yet well known. It has been in the QS phase for 7000 years and will stay there for another 500 years. It then will be a temporary L_5 Trojan of Venus for 700 years and then leave the co-orbital region. The object's eccentricity is large enough to bring it close to the Earth and frequent close approaches occur since the descending node stays close to the Earth's orbit. Although the Earth plays a significant role in its orbital evolution, all encounters are well outside its Hill sphere.

3.2. 3753 Cruithne

3753 *Cruithne* was the first asteroid to be recognized for being a co-orbital of the Earth (Wiegert et al., 1997). A detailed investigation of its motion can be found in Wiegert et al. (1998). At present it is in a compound HS–QS orbital configuration with the Earth (Namouni et al., 1999). The period is 770 years and the turning point of the compound orbit is 30° behind Earth. With its high inclination of $i \approx 20°$, it can pass through $\phi = 0$ without suffering a close approach with Earth, because both nodes are far from the Earth's orbit. It has been in its present state for about 500 years and will remain there for the next 5000 years. The distance of the nodes from Earth's orbit play a crucial role in its evolution as the libration mode is dependent on these: once the descending node is too far from Earth's orbit, the motion between 3753 *Cruithne* and Earth will decouple and it will circulate.

3.3. 2000 PH5

This asteroid is currently on its last horseshoe orbit with the Earth. At the time of epoch, it is trailing the Earth by 27° and will reach its next closest approach 100 years from now, leading the Earth by 27°. The semimajor axis will decrease from an average of $a = 1.0059$ to $a = 0.9922$, which is outside the horseshoe orbit region ($a = 0.9941$). Its orbit is well known (arc length of 1100 days) and has been observed by radar (Ostro et al., 2003).

4. 2001 CK32: 3753 *Cruithne's* twin

2001 CK32 is a Venus compound HS–QS orbiter on a very chaotic orbit. Its orbit is relatively well known with an observed arc length of 511 days and its motion is similar to that of 3753 *Cruithne*. Figure 1 shows the mean longitude difference of a sample orbit for the next 7000 years. At present the offset is at −50°. From the figure one observes that there is also some temporary QS motion and pure HS motion or compound motion with the offset at +50°. The mean period of the current compound trajectory is approximately 330 years. The clone trajectories are very chaotic, with a typical timescale of ∼ 110 years. Figure 2 plots the guiding center of the motion (Namouni, 1999). The horseshoe part of the motion is visible through the wedge-shaped curve centered around $\phi = 180°$, while the QS phase is illustrated by the ellipse centered around $\phi = 0°$.

The escape of the object occurs, as with 3753 *Cruithne*, when the nodes are farthest away from the orbit of the parent planet, which will happen 5000–6000 years from now: the motion between 2001 CK32 and Venus decouples and ϕ will circulate.

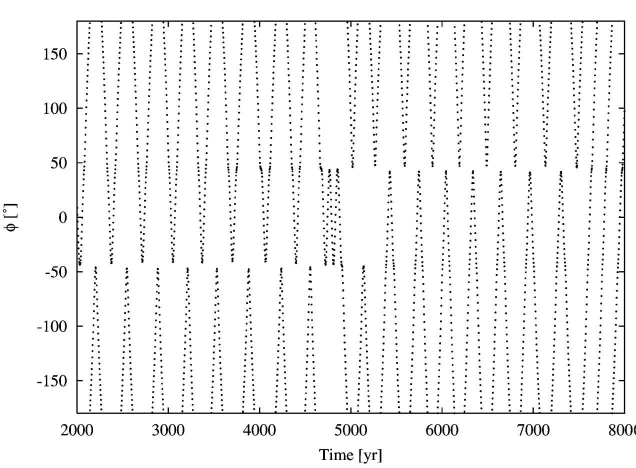

Fig. 1. The mean longitude difference $\phi = \lambda - \lambda_v$ of 2001 CK32 for the next 7000 years.

5. The orbit of 2002 AA29

Asteroid 2002 AA29, except for its inclination of 10.7°, has a low-eccentricity orbit that is similar to that of the Earth (Connors et al., 2002). The HS libration period is 190 years and the extrema of ϕ are $\phi_0 \approx 2.5°$ and $\phi_1 \approx 357.5°$, in agreement with Connors et al. (2002). It was further confirmed that the transitions between HS and QS motion occur near $t = 2600$ and $t = 3750$, each QS phase lasting 45 years. The period of libration while in the QS phase is 15 years. While Connors et al. (2002) list a few general properties of the motion, in this paper we study it in more detail. This is the only object that allows computation of its orbit for a significantly longer time than the other objects. Its Lyapunov time over this period is observed to be 450 years.

5.1. HS–QS transitions

There is a third HS–QS transition near $t = 6400$, after which further prediction of the motion becomes difficult.

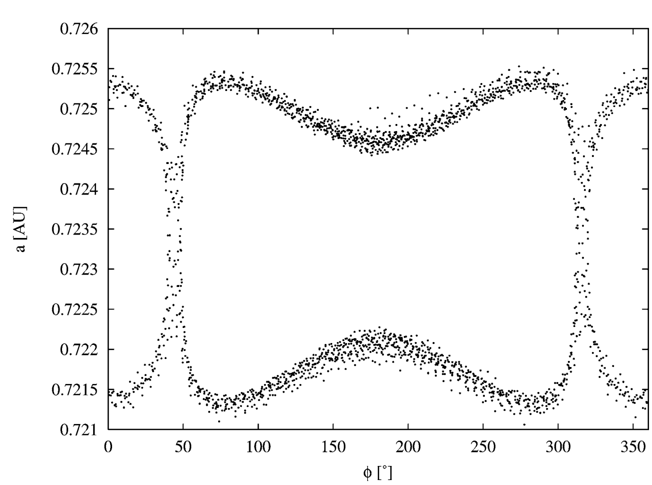

Fig. 2. The guiding center of 2001 CK32 for a sample orbit over a timespan of 5000 years. The HS and QS phases are clearly visible by the wedge-shaped curve around $\phi = 180°$ and the ellipse around $\phi = 0°$, respectively.

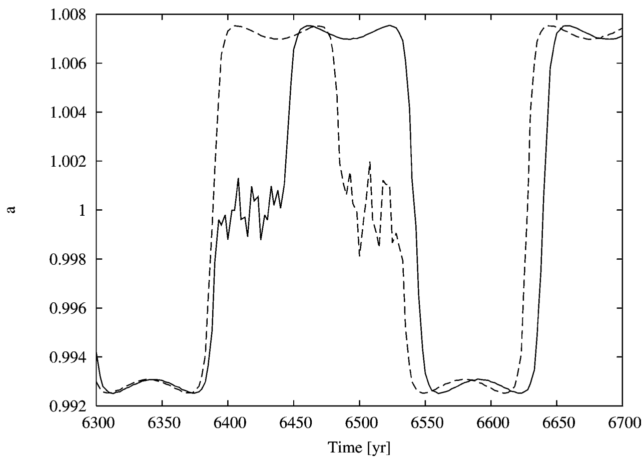

Fig. 3. The semimajor axis of 2002 AA29 for two neighboring clone orbits. One clone orbit (solid line) enters the QS phase from the trailing side, the other (dashed line) from the leading side half an HS period later.

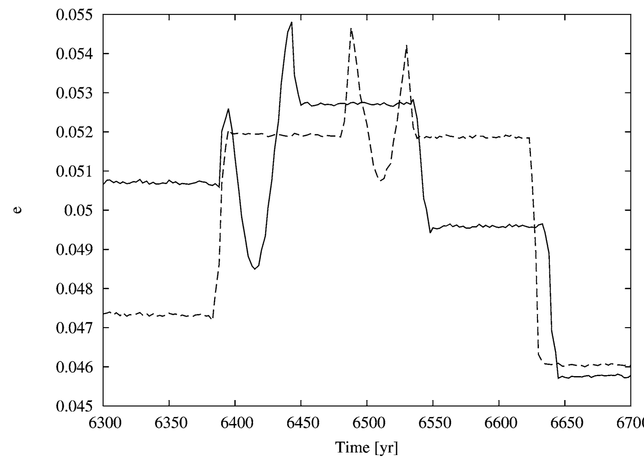

Fig. 4. Eccentricity of 2002 AA29 for two neighboring clone orbits around the third QS phase. The threshold value is $e \sim 0.05$ for the QS state to occur, as can be seen: the first orbit enters the QS while the other enters it later.

Going backwards in time, we confirmed the QS state starting at $t = 520$ found by Connors et al. (2002). The majority of clones entered the QS state at $t = 6400$ from the trailing side of the Earth. Most of the remaining orbits subsequently entered the QS state half a period later but a small sample did not. Hence the interval in which the orbit can be computed accurately is approximately $T \in (520, 6500)$. Therefore, we will restrict ourselves to the future evolution only.

The exact time and conditions when the next HS–QS transition occurs cannot be predicted beforehand. However, there seem to be certain conditions for these transitions to occur: at the transition the argument of pericenter, ω, is stationary and a maximum so that the transitions are dependent on the value of this parameter (Namouni, 1999). Additionally, the eccentricity needs to be larger than a certain threshold value. Only for the third transition, near $t = 6400$, is it possible to give a limiting value for the eccentricity which is $e_{\mathrm{QS}} > 0.0505$. Those orbits with $e > e_{\mathrm{QS}}$ when approaching the Earth from the trailing side enter the QS state. Those that do not continue on their HS orbit. Of that sample, only those with $e > e_{\mathrm{QS}}$ enter the QS state.

In Figure 3, the semimajor axis of two neighboring clone orbits in the interval $t = 6300$–6700 is depicted. One can see the first orbit, with the solid line, enters the QS state at $t = 6390$ while the other, with the broken line, does so half a period later. Figure 4 shows the eccentricity of these two orbits in the same interval. The shallow, quick dip observed in the eccentricity during this QS phase, the same as that observed by Namouni (1999) and Brasser et al. (2003), did not occur during the earlier QS phases, but is clearly visible this time and not unexpected.

5.2. *Transition conditions*

Although we have shown numerically that transitions between HS and QS motion occur regularly, the precise conditions cannot be established this way, so we attempt a mathematical derivation of the conditions for the HS–QS transition to occur. This is a qualitative extension to the work by Namouni et al. (1999), who explained the mechanism behind orbital transitions in quantitatively using plots of the ponderomotive potential for various values of e, i and ω. We treat ω as the independent variable. The eccentricity fluctuates over a large range while the inclination does not so that it can be assumed approximately constant. In our derivation we follow Namouni (1999).

The disturbing function describing the relative motion between the Earth and 2002 AA29 is given by

$$\mathcal{R} = -\frac{\mu n^2}{2\pi} \int_{-\pi}^{\pi} \Gamma^{-1/2}\, du, \tag{1}$$

where

$$\Gamma = -(e\cos u + a_{\mathrm{r}})^2 + 4\left(\sin\frac{1}{2}\phi - e\sin u\right)^2 + 4s^2\sin^2(u+\omega) \tag{2}$$

with $u = nt - \bar{\omega}$, $s = \sin(1/2)i$ and $a_{\mathrm{r}} = a - a_{\mathrm{e}}$, where a_{e} is the semimajor axis of the Earth, n is the mean motion, $\bar{\omega} = \Omega + \omega$ with Ω being the longitude of the ascending node; the quantity Γ is the square of the distance between the two bodies. The elements e, ω and i are technically relative elements. Since the eccentricity and inclination of 2002 AA29 are larger than those of the Earth itself, it is safe to assume, at least for the inclination, that they are the elements of the asteroid. Over the reliable computation time, ω_{e} stays close to 180° so that the relative argument of pericenter may be replaced by that of 2002 AA29 in the last term.

The first future HS–QS transition has $e = 3.4\varepsilon$, $i = 15.6\varepsilon$, $\phi = 7.9\varepsilon$ and $a_{\mathrm{r}} = 0.8\varepsilon$; $\varepsilon = 0.01$ for the Earth, so that we can safely ignore the contribution due to a_{r}. Namouni (1999) expands $\mathcal{R}$ in terms of ϕ^{-1} down to ϕ^{-3} (Eq. (34)). This is insufficient to explain the observed transitions because we know these are clearly dependent on ω. We therefore expand

$\mathcal{R}$ to ϕ^{-5}, resulting in

$$\mathcal{R}_1 = -\frac{1}{2}\csc\frac{1}{2}\phi - \frac{1}{32}\csc^3\frac{1}{2}\phi\left(7e^2 - 5s^2\right) - \frac{1}{2048}\csc^5\frac{1}{2}\phi\left[297e^4 - 720e^2s^2 + 144s^2 - 480e^2s^2\cos(2\omega)\right] + O\left(\phi^{-7}\right), \tag{3}$$

where we have the dependence on ω in the last term. Equation (3) may only be used for values of $\phi > \phi_c = 8\varepsilon^3/a_0^2$ because at $\phi = \phi_c$, Eq. (1) has a maximum: for $\phi < \phi_c$, 2002 AA29 is in the QS phase and for $\phi > \phi_c$, it is in the HS phase. For this transition $\phi_c \approx 2.2°$. From Namouni (1999) we know that

$$a_r^2 = a_0^2 - \frac{8}{3}\mathcal{R}, \tag{4}$$

where a_0 is related to the hamiltonian (Namouni, 1999) and

$$\dot{\phi} = -\frac{3}{2}a_r + \frac{\partial\mathcal{R}}{\partial a_r}. \tag{5}$$

In fact, Eq. (4) is the same as Eq. (13) of Namouni (1999) if Eqs. (1) and (2) are applied. It appears that a_0 can be interpreted as the maximum half-width of the HS orbit, which for 2002 AA29 is 0.08 AU. Hence, from Eqs. (3) and (5) and ignoring the contribution due to a_r, we have $(\partial\mathcal{R})/\partial a_r = 0$ so that $\dot{\phi}$ is dependent on $\cos(2\omega)$. This implies that the turning points of ϕ, i.e., the limits of the HS orbit, depend on ω, so that the distance of closest approach at the end of each HS cycle to the Earth is a maximum when $\omega = 90°$. These variations in the approach distance are depicted in Fig. 5, which is a plot of ϕ vs time in the ϕ-range $-5°$ to $5°$. This should be compared with Fig. 6 which depicts ω vs time: one can see that the minima of $|\phi|$ are farther from the Earth when ω is closest to 90°.

From Namouni (1999) $\dot{\omega} > 0$ during the HS phase and during the QS phase $\dot{\omega} < 0$. Averaging Eq. (3) with respect to ϕ for the HS phase one finds that

$$\bar{\mathcal{R}} = \int_{\phi_0}^{\phi_1} \mathcal{R}\,d\phi \propto \csc\frac{1}{2}\phi + \csc^3\frac{1}{2}\phi + \csc^5\frac{1}{2}\phi\cos(2\omega), \tag{6}$$

so that $\bar{\mathcal{R}}$ has a minimum at $\omega = 90°$. Furthermore $\dot{\omega}$ depends on $-\csc^5(1/2)\phi\cos(2\omega)$, which explains the rapid increase in ω with each approach to the Earth as well as why the long-term average of $\dot{\omega}$ is a maximum when $\omega = 90°$.

The motion of 2002 AA29 is as follows: The value of $\bar{\mathcal{R}}$ is very sensitive to the integration limits of ϕ as well as to e and i. By inserting the initial conditions of our computations into Eq. (1), we find that ω is limited to a range $\omega \in [\omega_0, \omega_1]$, centered around $\omega = 90°$, with both ω_0 and ω_1 being dependent on e, i and the limits of ϕ. Once $\omega > 90°$, 2002 AA29 is forced to experience closer approaches to Earth at the end of the HS cycle in order to maintain the condition $\dot{\omega} > 0$. As $\omega \to \omega_1$, the approach to Earth to satisfy $\dot{\omega} > 0$ is so close that 2002 AA29 is able to cross the effective potential maximum at $\phi = \phi_c$ and fall into the QS phase. Now $\dot{\omega} < 0$ and ω regresses until the asteroid can cross the potential barrier at $\phi = \phi_c$ again. By now $\omega < 90°$, 2002 AA29 is back in the HS phase, $\dot{\omega} > 0$ and will increase until it reaches its maximum at $\omega = 90°$. The approach distance will be a maximum at this time and the cycle repeats itself.

5.3. *The influence of other planets*

While trying to understand the HS–QS transition mechanism, we experimented with the influence of the other planets of the Solar System by including or excluding them from our numerical simulations. To our surprise, the first HS–QS transition did not occur when we simulated the system with only the Sun, Earth and 2002 AA29. Including either one of Jupiter or Venus, which are the main perturbers of Earth's orbit (see, e.g., Brétagnon, 1974), also gave negative results. When both Venus and Jupiter were included, the transition did occur. Including only the giant planets also produced the transition, though it left the QS

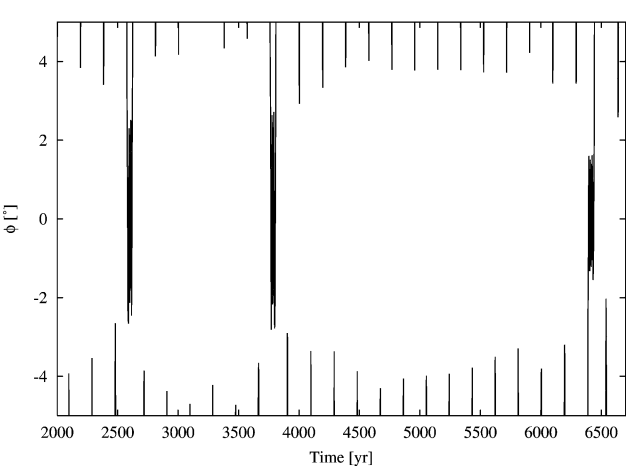

Fig. 5. A plot of ϕ vs time centered around $\phi = 0$. The comblike portions of the trajectory appearing at the top and the bottom are offset by half a libration period and represent close approaches to the Earth at the end of each HS cycle. The angle ϕ remains near zero during the QS phases at 2600, 3750 and 6400 AD. The variation in the approach distance is clearly visible.

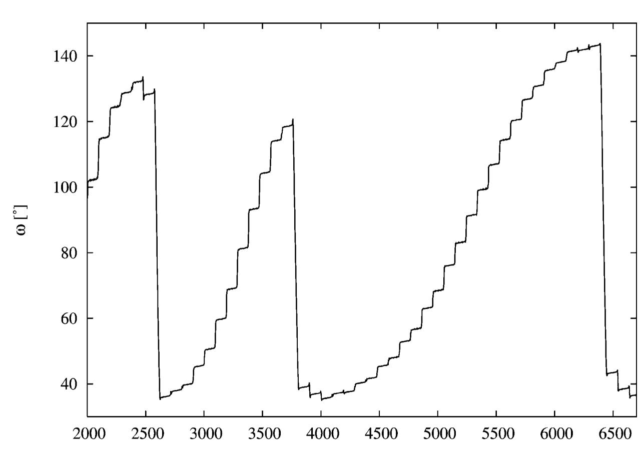

Fig. 6. ω of 2002 AA29 for one of the orbits. It appears to be librating.

phase half a period later. What happens is this: The influence of the other planets determines ω_e and hence $\omega - \omega_e$ (Wiegert et al., 1998), which should be the variable used in Eq. (3) to determine the transition conditions. The transition is a 3-body problem, but conditions for the transition depend on the influence of the other planets (Christou, 2000).

6. 2001 GO2: Earth HS 2

The trajectory of 2001 GO2 enables it to have frequent transitions between the HS and QS states. The extrema of ϕ are $\phi_1 \approx 18°$ and $\phi_2 \approx 342°$. The period is 190 years: the same as 2002 AA29. The orbit of 2001 GO2 is chaotic and has an observed arc of only five days, so that the following results need to be interpreted with some caution. The Lyapunov time for this object is about 180 years.

The divergence between neighboring orbits is accelerated during the first HS–QS transition at $t = 2201$. These QS phases occur frequently and the majority of the orbits do enter the first QS phase; each is consistently for one QS period only and lasts 45 years. The average amplitude of ϕ during the QS phase is $\Delta\phi \approx 36°$. The condition for the asteroid to enter the first QS phase is $i \geqslant 4.970° \pm 0.003°$ in the barycentric frame. The inclination in the ecliptic frame is given by

$$\cos i_f = \cos(i - i_e) - 2\cos i \cos i_e \sin^2 \frac{1}{2}(\Omega - \Omega_e), \qquad (7)$$

where i_f is the inclination in the ecliptic frame, i_e is the inclination of the Earth in the barycentric frame and Ω_e is the node of the Earth in the barycentric frame. We found no dependency on either the eccentricity or the argument of pericenter. Entering the QS phase always happens from the trailing side of the Earth and every other complete libration, as shown for a sample orbit in Fig. 7. The time of escape from the co-orbital region depends on whether or not the object enters the first QS phase. For those clones that do not, the time of escape lies between 500–700 years from now. For those that do, the time of escape is between 2000–2500 years.

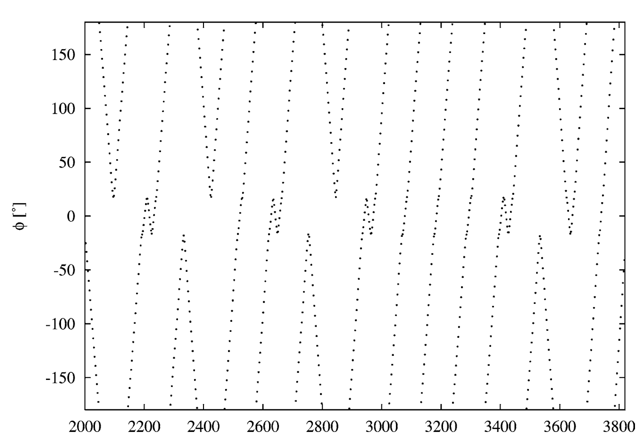

Fig. 7. The ϕ-trajectory of a sample orbit. Note the frequent transitions to the QS phase, which happen about every 400 years or so.

7. 2003 YN107: an Earth QS in action

The Earth co-orbital 2003 YN107 is at present a QS of the Earth (see Connors et al., 2004).

7.1. Overview

Its orbit can be accurately computed for 250 years in the past and about 115 years into the future. At $t = 1750$ it experienced a chaotic HS–QS transition. Going forward in time, a divergence occurs in the eccentricity and inclination after completing half a horseshoe period at $t = 2066$, so that after the next approach at $t = 2120$ further simulations become meaningless. Most likely the object will circulate afterwards (Connors et al., 2004). Like 2001 GO2, the transitions appear to depend on the inclination only, although the motion is similar to that of 2002 AA29. The HS phase has a period of 133 years, significantly shorter than 2002 AA29 and 2001 GO2. The extrema of ϕ vary slightly but are approximately $\phi_0 = 1.5°$ and $\phi_1 = 358.5°$, so it librates with a larger amplitude than 2002 AA29 and gets significantly closer during each encounter (about 0.026 AU vs 0.039 AU for 2002 AA29).

7.2. The QS phase

The elements in Table 1 are taken during the QS phase and thus change rapidly. 2003 YN107 entered the QS phase with an eccentricity of $e = 0.045$ and a barycentric inclination of $i = 5.81°$. During the current phase the argument of pericenter and node regress at a uniform rate of $\dot{\omega} \approx -9.1°\ \mathrm{yr}^{-1}$ and $\dot{\Omega} = (1/8)\dot{\omega}$ (Namouni, 1999). The eccentricity decreases also at a uniform, linear rate, which

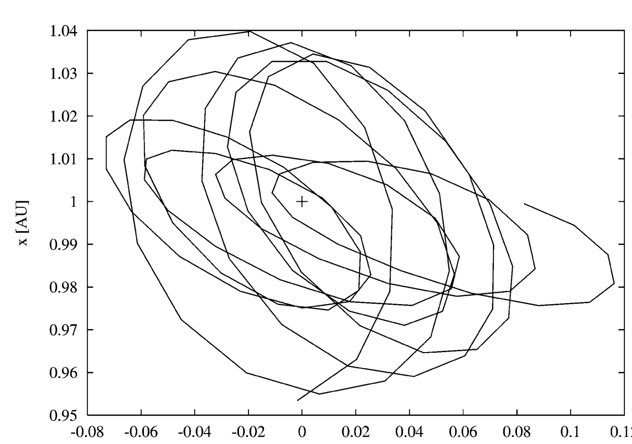

Fig. 8. Side view of the trajectory of 2003 YN107 during its QS phase. The Earth is centered on (0, 1) and marked with a +.

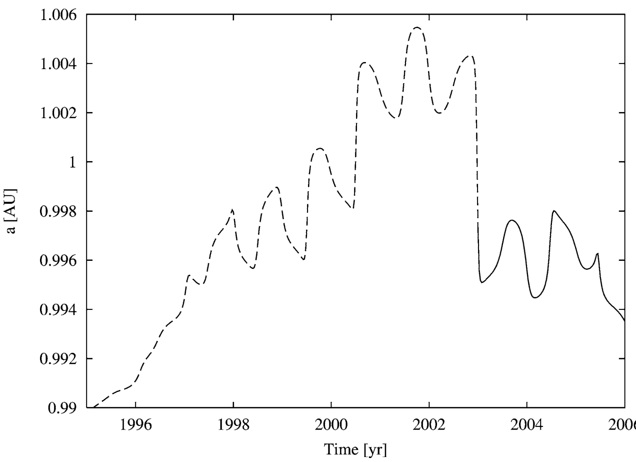

Fig. 9. The semimajor axis of 2003 YN107 for the time interval 1995 to 2006.

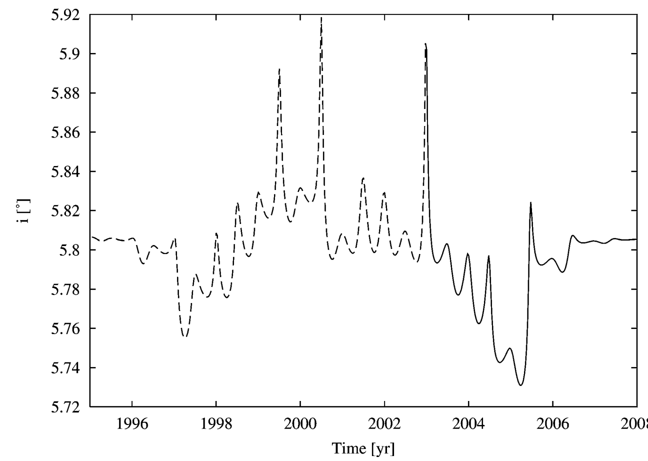

Fig. 10. The barycentric inclination of 2003 YN107 for the time interval 1995 to 2008. Note the large fluctuations with period half a year.

implies that, locally, 2003 YN107 exchanges angular momentum with the Earth during the whole QS phase. The inclination before and after the QS phase is the same.

Figure 8 shows the motion of 2003 YN107 during the QS phase. The coordinate system is geocentered and rotating. The Earth is marked with a + sign. The loops are caused by the eccentricity of YN107 relative to that of the Earth and are completed in one year. The longitudinal oscillation of the loops is a long-term effect with a period of about ten years. Figure 9 depicts the semimajor axis in the interval $t = 1995$ to $t = 2006$. The fact that for a significant fraction of the QS phase $a > 1$ implies that 2003 YN107 stays on average farther away from the Sun than the Earth while it also lingers in the neighborhood of the Earth itself.

The inclination fluctuates strongly during the QS phase with an average period of about half a year. These fluctuations are shown in Fig. 10. The half-year periodicity arises from the nonaveraged disturbing function $\Gamma^{-1/2}$. Since

$$\frac{di}{dt} \propto \frac{\partial \Gamma^{-1/2}}{\partial \omega} \tag{8}$$

we have $di/(dt) \propto \cos(2u + 2\omega)$, which results in the half-year periodicity.

8. Summary and conclusions

We have analyzed the orbits of several new co-orbital asteroids in the inner Solar System as well one already known. These are 2001 CK32, a Venus co-orbital whose orbit is similar to 3753 *Cruithne*, 2001 GO2, a second Earth HS companion, and 2002 AA29 and 2003 YN107. The orbit of 2001 CK32 is a compound one, with a libration period of 330 years and is highly chaotic. The orbits of 2002 AA29, as well as 2001 GO2, both have a libration period of 190 years (see Connors et al. (2002) about 2002 AA29). Both have a small eccentricity and enter the QS phase regularly. Each of these QS phases last 45 years. 2003 YN107 is currently an Earth QS, which lasts around 9 years from 1997 to 2006, completing one QS period only. Using new theoretical insights based on those of Namouni (1999) and Namouni et al. (1999), and our own simulations, we derive conditions for the HS–QS transitions of 2002 AA29 to occur. These results can readily be applied to 2001 GO2 and 2003 YN107.

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