A MINIMUM HYPOTHESIS EXPLANATION FOR AN IMF WITH A LOGNORMAL BODY AND POWER LAW TAIL

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Abstract We present a minimum hypothesis model for an IMF that resembles a lognormal distribution at low masses but has a distinct power-law tail. Even if the central limit theorem ensures a lognormal distribution of condensation masses at birth, a power-law tail in the distribution arises due to accretion from the ambient cloud, coupled with a non-uniform (exponential) distribution of accretion times.

1. A Model for the Initial Mass Function

Observations of the field star IMF have long established the existence of a power-law tail in the intermediate and high mass regime (Salpeter 1955). Recent observations of stars within young embedded clusters (e.g., Muench et al. 2002), have also established the existence of a low mass peak in the stellar mass distribution. Submillimeter observations of dense protostellar condensations (e.g., Motte, André, & Neri 1998) also imply a power-law tail in the intermediate and high mass regime. Given the evidence for a peaked distribution, a natural explanation is to invoke the central limit theorem of statistics and argue that the IMF should be characterized by the lognormal probability density function (pdf)

$$f(m) = \frac{1}{\sqrt{2\pi}\sigma m} \exp\left[-\frac{(\ln m - \mu)^2}{2\sigma^2}\right] \tag{1}$$

for the masses m, where μ and σ^2 are the mean and variance of $\ln m$. Now, assume that condensation masses are initially drawn from the above distribution with mean μ_0 and variance σ_0^2. Furthermore, if they accrete mass at a rate $dm/dt = \gamma m$, (this may be a reasonable assump-

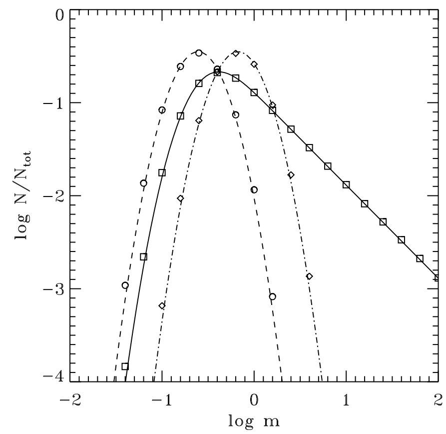

Figure 1. Evolution of a distribution of masses m undergoing accretion growth $dm/dt = \gamma m$. Dashed line and circles: an initial lognormal probability density function (pdf), with $\mu_0 = -1.40, \sigma_0 = 0.52$. Dash-dotted line and diamonds: pdf after accretion growth of all masses for a time $t = \gamma^{-1}$. Solid line and squares: pdf if accretion lifetimes have a pdf $f(t) = \delta e^{-\delta t}$ and $\delta = \gamma$. From Basu & Jones (2004).

tion at the condensation stage, but not for a protostar accreting from its parent core), and the accretion time t has an exponential distribution with a pdf $f(t) = \delta e^{-\delta t}$, the pdf of final masses is

$$f(m) = \frac{\alpha}{2} \exp\left[\alpha\mu_0 + \alpha^2\sigma_0^2/2\right] m^{-1-\alpha} \times \mathrm{erfc}\left[\frac{1}{\sqrt{2}}\left(\alpha\sigma_0 - \frac{\ln m - \mu_0}{\sigma_0}\right)\right]. \quad (2)$$

In this equation, $\alpha = \delta/\gamma$ is the dimensionless ratio of 'death' rate to 'growth' rate of condensations, and erfc is the complementary error function. This analytically derivable three-parameter formula has the advantage of being near lognormal at low masses but having an asymptotic dependence $f(m) \propto m^{-1-\alpha}$. If the 'death' and 'growth' rates are both controlled by the parent cloud, we might expect $\alpha \approx 1$, so that the distribution is Salpeter-like. See Basu & Jones (2004) for details, and references to other areas (e.g., distribution of incomes, city sizes, and Internet file sizes) where similar ideas are applicable.

References

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